MULTI-RESOLUTION LEVEL SET IMAGE SEGMENTATION USING WAVELETS

Fares S. Al-Qunaieer, Hamid R. Tizhoosh

Systems Design Engineering
University of Waterloo
Waterloo, ON N2L 3G1, Canada
falqunai@engmail.uwaterloo.ca,
tizhoosh@uwaterloo.ca

Shahryar Rahnamayan

Electrical and Computer Engineering
University of Ontario Institute of
Technology, 2000 Simcoe Street North
Oshawa, ON L1H 7K4, Canada
shahryar.rahnamayan@uoit.ca

ABSTRACT

Level set methods have been used for image segmentation. Because partial deferential equations are solved to propagate a curve, level-set image segmentation has a slow convergence speed. The objective of this paper is to propose a method that increases the convergence speed. The proposed approach exploits the benefit of multi-resolutional analysis. Wavelet transform is used to decompose the image into different resolutions. The obtained results show a great improvement in terms of speed and accuracy.

Index Terms— segmentation, level sets, multi-resolution

1. INTRODUCTION

Image segmentation is considered one of the most important steps in image processing applications. The segmentation results are used for various purposes, such as feature extraction, object analysis and recognition. There are numerous algorithms for image segmentation. Each of these algorithms is depending on some criteria, such as discontinuity (edges), textures and colors. Active contour image segmentation has received a lot of attention in recent years. A curve is propagated to the edges of the desired object(s). There are mainly two types of active contours, parametric and geometric. In parametric implementation, explicit points are defining the contour, while it is defined implicitly by a higher dimensional function in geometric active contours. Snakes [1] considered to be the most famous parametric active contours. Geometric level-set method was introduced by Osher and Sethian [2] for interface propagation. It has been used for many applications, such as computational fluid mechanics, computer graphics, shape optimization, inverse problems, and image analysis. A contour is implicitly defined by using a Lipschitz-continuous function $\phi(x, y)$ called level set function. Usually, the contour is defined at the zeroth level of $\phi(x, y)$ such that positive and negative signs represent two different regions. Caselles et al. [3], proposed a geometric active contour based on level set formulation to solve the segmentation problem. A similar work was proposed by Malladi et al. [4, 5]. Geodesic active contours was proposed by Caselles et al. [6, 7]. The objective is to find geodesic (minimum distance) curves in a Riemanian space with metric derived from the image content. Active contours without edges, which is the most common region-based level set algorithm, was proposed by Chan and Vese [8]. The authors proposed a level set method to minimize Mumford-Shah segmentation model [9] for piecewise constant approximation of the image. Later, a piecewise smooth approximation was proposed by Vese and Chan [10]. Similar active contour models and formulations were proposed by Yezzi et al. [11] and Tsai et al. [12]. Level set formulation of active contour uses partial differential equations for curve propagation resulting in a slow propagation. For this reason, active contours are not applicable for some real-time applications. We propose a multi-resolution approach based on the wavelet transform. By utilizing coarse resolutions for curve initialization, convergence speed can increase rapidly. In addition to the decrease of execution time, the proposed method seems to be robust for noisy images.

The remaining of the paper is organized as follows: the problem of level set segmentation is formulated in Section 2. The proposed approach is described in Section 3. In Section 4, experiments and results are presented. Section 5 concludes the paper.

2. PROBLEM FORMULATION

Given an input image I, the objective of image segmentation is to partition I into n regions, such that $\bigcup_{j=1}^{n} I_j = I$, I_j is a connected set, where $j = 1, 2, \ldots, n$, and $I_j \cap I_k = \phi$ for all j and k, where $j \neq k$. Let C represent a contour in the image domain, which is defined as

$$C = \{(x, y) : \phi(x, y, t) = 0\}, \forall (x, y) \in \Omega, \qquad (1)$$

where Ω denotes the entire image domain, $\phi(x, y, t)$ is the level set function, and t is the time through deformation. The evolution of the curve is performed implicitly as the level set function evolve. Chan and Vese [8] proposed a level set method to minimize Mumford-Shah segmentation model [9] for piecewise constant approximation of the image. The level set formulation is defined as

$$\begin{aligned} E(c_1, c_2, \phi) = \mu \int_\Omega \delta(\phi(x,y))|\nabla\phi(x,y)|\, dxdy \\ +\upsilon \int_\Omega H(\phi(x,y))\, dxdy \\ +\lambda_1 \int_\Omega |I(x,y) - c_1|^2 H(\phi(x,y))\, dxdy \\ +\lambda_2 \int_\Omega |I(x,y) - c_2|^2 (1 - H(\phi(x,y)))\, dxdy, \end{aligned} \tag{2}$$

where I is the input image, μ, υ, λ_1 and λ_2 are fixed parameters (> 0), and c_1 and c_2 are averages of the areas inside and outside curve C, respectively. $H(x)$ is the Heaviside function, defined by

$$H(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}, \tag{3}$$

and $\delta(x)$ is Dirac measure defined as $\frac{d}{dx}H(x)$.

3. PROPOSED APPROACH

Although active contour methods provide good solution for image segmentation, they suffer from a serious drawback. They are computationally expensive resulting in slow convergence. This is due to the numerical approximation that solves partial differentiation equations. Several attempts were performed to solve this problem, such as narrow band [5]. Multi-resolution techniques (such as pyramid multi-resolution analysis, scale-space representation, and wavelets) have been applied to enhance active contours. Most proposed methods were designed for parametric snakes. Dehmeshki et al. [13] improved snake algorithm by using wavelets. The contour is initialized at the coarsest resolution of the image. The initial curve of finer resolution is the converged one of the previous coarse resolution. Similar algorithm was proposed by Yoon et al. [14] to improve Gaussian of Gradient Force (GGF) snake. The authors reported speed increase with high accuracy. Curvelets are used to enhance geodesic active contours as well [15]. The edge map was obtained using curvelet thresholding. Initialization was performed in the coarsest resolution, and for each subsequent finer resolution, the level set function ϕ is defined as the converged level set function of the previous resolution. Wavelets is a tool for image processing and can suppress the noise in an image in the coarse scales. Also, at the coarse levels, weak edges are avoided and the contour is attracted to strong edges only. If the initialization of the curve begins with a coarse version of the image, the convergence will be fast because many details are omitted and only the main features and edges remain. This enables us to implement a much faster algorithm. To the best of our knowledge, there is not any level set segmentation algorithm that uses wavelets multi-resolution curve propagation except for the mentioned curvelet algorithm. The objective of the proposed approach is to increase the speed of level set segmentation. This is achieved by starting from the coarsest resolution of image. This algorithm is general and can be used with any level set segmentation. The algorithm is presented in Algorithm 1. I is the input image. Number of resolutions r is provided by the user. The image I is decomposed into r levels of resolutions using wavelet transform. The results of wavelet decomposition are approximation coefficients (A), horizontal details (H), vertical details (V) and diagonal details (D). The original image is stored in I_r and approximation coefficients A_i is stored in I_i, where $i = 1, \ldots, r-1$. Starting from coarsest image resolution I_1, the level set algorithm of choice is run on I_i. As the current resolution is not of the original image, the converged curve of the current resolution is up-sampled to the finer resolution. Up-sampling is performed by a factor of 2. The algorithm continues until the curve is converged in the original resolution.

Algorithm 1 Proposed approach

```
I = input image
r = number of resolutions
Wavelet Decomposition
[A_k, H_k, V_k, D_k] = Wavelet_r(I), k = 1, ..., r − 1
I_r = original image
for i = r − 1 to 1 do
    I_i = A_i
end for
Multi-resolution Propagation
for i = 1 to r do
    if i = 1 then
        Initialize curve on I_i
    else
        initial curve ← up-sampled curve (previous iteration)
    end if
    Run level set algorithm on I_i
    if i < r then
        Up-sample converged curve to the (i + 1)th resolution
    end if
    i = i + 1
end for
```

4. EXPERIMENTS AND RESULTS

The level set algorithm utilized in these experiments is the region-based level set algorithm proposed by Chan and Vese [8]. Haar wavelet [16] is used for image decomposition due to its simplicity. For each image, a decomposition of three levels of resolutions is preformed. The evolution of the curve will be stopped if there is a change less than ϵ in five consecutive iterations, or if the number of iterations exceeds 5000. The threshold value ϵ should be very small to stop evolution.

The initialization of the curve is performed as multiple circles; Vese and Chan [10] had shown that this type of initialization is efficient for region-based level set algorithms. Two experiments were conducted in this section. The segmentation result of each image is compared with ChanVese algorithm: 1) A set of 7 different images (Various image set). Each image has different characteristics, such as level of noise and type of boundary whether it is sharp or fuzzy, 2) A set of 7 synthetic prostate images (Synthetic image set). The objective is to test the performance of the algorithm for noise corrupted case. Both Various and Synthetic image sets have manually segments called "gold standard images". The numerical results of the proposed algorithm, WChanVese, and the original ChanVese algorithm consist of the execution time, accuracy and number of iterations in each level of resolution. Time is measured for the complete execution of the two algorithms including the initialization phase. For WChanVese algorithm, the number of iterations in each level of resolution is counted. Because ChanVese algorithm is run in only one resolution, the iterations of one level are counted. Two metrics are used to calculate accuracy, namely, dice coefficient and Jaccard index:

$$Dice = \frac{2|I_n \cap I_G|}{|I_n| + |I_G|}, \tag{4}$$

$$Jaccard = \frac{|I_n \cap I_G|}{|I_n \cup I_G|}, \tag{5}$$

where I_n is the segmented image, and I_G is the gold image.

Results – The first experiment is conducted using Various image set (Table 1). It can be seen that for all images WChanVese converged faster than ChanVese, with nearly double speed for some images. In WChanVese, the iterations in the original resolution (iterations 3) are far less than in ChanVese. Even with slightly higher number of iterations, as for shadow image, it converges slightly faster because the starting position of the third stage in WChanVese is better than the starting position in ChanVese. The accuracies of the two algorithms are very close, where WChanVese overall performance is slightly better. Very low standard deviation indicates more reliable results. For the second experiment, Synthetic image set is used (Table 2). It can be noticed that there are huge improvements with both accuracies and execution time when using WChanVese. The proposed algorithm proved to obtain better results for noisy images. ChanVese algorithm didn't converge for any of the images, while WChanVese converged very fast for all of them. The accuracy of WChanVese was much better for all images with good consistency as indicated by the very small standard deviations of dice coefficients and Jaccard index. Fig. 1 illustrates some samples of the segmentation results.

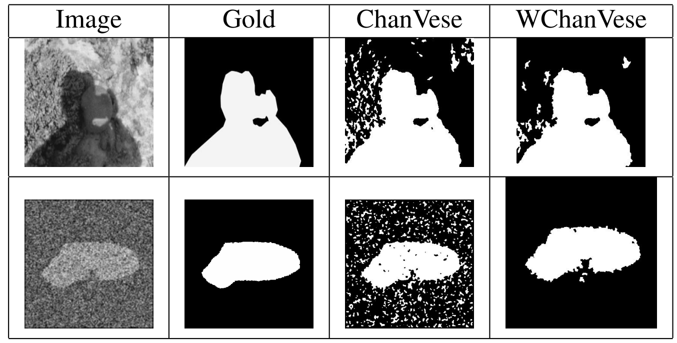

Fig. 1. Sample results from Various and Synthetic image sets.

5. CONCLUSION

In this paper, a multi-resolution algorithm based on wavelet decomposition was proposed to reduce the time of curve propagation in level set segmentation. This research intend to solve the slow convergence problem of level set algorithm and make it applicable for real-time applications. The experimental results are promising. The improvement of runtime is obvious. Also, accuracy of segmentation in noisy images are greatly improved. Other types of wavelets, such as Daubechies or Morlet wavelets can be used. Also, a method to select the optimal number of resolutions for wavelet decomposition shall be investigated.

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	ChanVese				WChanVese					
Image	Dice	Jaccard	Time (s)	Iterations	Dice	Jaccard	Time (s)	Iter1	Iter2	Iter3
block	0.9571	0.9178	5.16	138	0.9281	0.8659	3.05	46	44	86
fleck	0.8689	0.7682	3.16	177	0.8925	0.8059	1.16	71	24	63
potat	0.9730	0.9475	5.07	90	0.9703	0.9423	4.02	36	45	67
rad	0.9805	0.9618	4.02	74	0.9812	0.9632	2.71	36	28	46
shadow	0.8989	0.8163	5.11	142	0.9462	0.8979	4.80	71	47	152
stones	0.9716	0.9448	5.02	112	0.9767	0.9546	3.11	65	63	61
zimba	0.9797	0.9601	3.57	110	0.9682	0.9383	2.17	32	37	62
μ	0.9471	0.9024	4.44	-	0.9519	0.9097	3.00	-	-	-
σ	0.0447	0.0778	0.8438	-	0.0322	0.0571	1.87	-	-	-

Table 1. Results of Various image set. Dice coefficient and Jaccard index measure the accuracy, and iterations is the number of iterations until convergence. Iter1, Iter2 and Iter3 are iterations of coarsest, middle and original resolutions.

	ChanVese				WChanVese					
Image	Dice	Jaccard	Time (s)	Iterations	Dice	Jaccard	Time (s)	Iter1	Iter2	Iter3
1	0.6763	0.5109	408.46	5000	0.9279	0.8655	7.45	129	96	92
2	0.7372	0.5838	434.06	5000	0.9473	0.8999	10.46	243	60	151
3	0.7482	0.5976	425.79	5000	0.9590	0.9213	6.44	119	37	65
4	0.7859	0.6473	399.75	5000	0.9576	0.9187	7.53	85	37	122
5	0.7432	0.5914	412.20	5000	0.9402	0.8872	9.67	158	57	142
6	0.7779	0.6365	393.97	5000	0.9607	0.9244	4.73	103	37	68
7	0.7678	0.6231	436.08	5000	0.9478	0.9007	6.77	95	95	106
μ	0.7481	0.5987	415.76	-	0.9486	0.9025	7.58	-	-	-
σ	0.0365	0.0454	16.56	-	0.0118	0.0212	1.95	-	-	-

Table 2. Results of Synthetic image set. Dice coefficient and Jaccard index measure the accuracy, and iterations is the number of iterations until convergence. Iter1, Iter2 and Iter3 are iterations of coarsest, middle and original resolutions.

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