ARTICLE IN PRESS

Swarm and Evolutionary Computation xxx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

Swarm and Evolutionary Computation

journal homepage: www.elsevier.com/locate/swevo

Opposition based learning: A literature review

Sedigheh Mahdavi[a], Shahryar Rahnamayan[a,*], Kalyanmoy Deb[b]

[a] Department of Electrical, Computer, and Software Engineering, University of Ontario Institute of Technology (UOIT), Oshawa, Canada
[b] Department of Electrical and Computer Engineering, Michigan State University, East Lansing, USA

ARTICLE INFO

Keywords:
Opposition-based computation
Soft computing
Evolutionary computation
Artificial neural network
Reinforcement learning
Opposition schemes
Machine learning
Computation

ABSTRACT

Opposition-based Learning (OBL) is a new concept in machine learning, inspired from the opposite relationship among entities. In 2005, for the first time the concept of opposition was introduced which has attracted a lot of research efforts in the last decade. Variety of soft computing algorithms such as, optimization methods, reinforcement learning, artificial neural networks, and fuzzy systems have already utilized the concept of OBL to improve their performance. This survey has been conducted on three classes of OBL attempts: a) theoretical, including the mathematical theorems and fundamental definitions, b) developmental, focusing on the design of the special OBL-based schemes, and c) real-world applications of OBL. More than 380 papers in a variety of disciplines are surveyed and also a comprehensive set of promising directions are discussed in detail.

1. Introduction

The concepts of opposition widely exist in the world around us, but it has sometimes been understood in different ways. For instance, opposite particles in physics, antonyms in languages, complement of an event in probability, antithetic variables in the simulation, opposite proverbs in the culture, absolute or relative complement in the set theory, subject and object in the philosophy, good and evil in animism, opposition parties in politics, theses and antitheses in dialectic, and dualism in religions and philosophies. It seems that the explanation of different entities becomes a tough task without using the concept of opposition such as the east-west, south-north, and hot-cold which cannot be described separately [216,205].

Opposition-Based Learning (OBL) is a novel research field which has already attracted a recognizable interest in the past decade. Many soft computing algorithms have been enhanced by utilizing the concept of OBL such as, Reinforcement Learning (RL), Artificial Neural Networks (ANN), Fuzzy Systems, and variant optimization methods such as Genetic Algorithms (GA) [97], Differential Evolution (DE) [202,274], Particle Swarm Optimization (PSO) [121,122], Biogeography-based Optimization (BBO) [267], Harmony Search (HS) [96], Ant Colony System (ACS) [66,67], Gravitational Search Optimization (GSO), Group Search Algorithm (GSA), Artificial Bee Colony (ABC) [116], Simulated Annealing (SA), etc. In 2005, the fundamental concept of OBL [297] was proposed which considers the current estimate (guess) and its corresponding opposite simultaneously to find a solution efficiently. When the main goal of an algorithm is finding the optimal solution for an objective function, considering an estimate and its opposite simultaneously can be beneficial to enhance the performance of the algorithm. The advantages of applying the OBL concept have been investigated to define the transfer function and weights of neural networks, creating candidate solutions of evolutionary algorithms, and action policy of reinforcement agents.

Since January 2005, more than 400 publications have been published on the OBL concept. These research works have been published in conferences, journals and books which are in machine learning or soft computing. Among these papers, 60% are journal papers, 38% are conference papers, and 2% books/thesis. Fig. 1 shows the number of publications and citations per year obtained by the website, Thomson Reuters (formerly ISI) Web of Knowledge. Two surveys on OBL have been published in [7,359] which surveyed 52 and 138 papers but they do not covered many research works which were published in the recent years. Therefore, based on the fast progress of research works on OBL and its applications in the science and engineering fields, it is motivated us to prepare an up to date comprehensive survey including the latest theoretical and developmental researches and promising future directions of OBL. This paper attempts to provide a global overview of research works on OBL from different perspectives. Some of research works focus on the mathematical proofs and theoretical definitions to investigate and use the benefits of OBL; some are on the special developments for various schemes of using OBL in the machine learning methods; and others are on the different applications of OBL in the various science and engineering applications such as power systems, pattern recognition

* Corresponding author.
E-mail addresses: Sedigheh.Mahdavi@uoit.ca (S. Mahdavi), Shahryar.Rahnamayan@uoit.ca (S. Rahnamayan), kdeb@egr.msu.edu (K. Deb).

http://dx.doi.org/10.1016/j.swevo.2017.09.010
Received 13 November 2016; Received in revised form 12 September 2017; Accepted 16 September 2017
2210-6502/

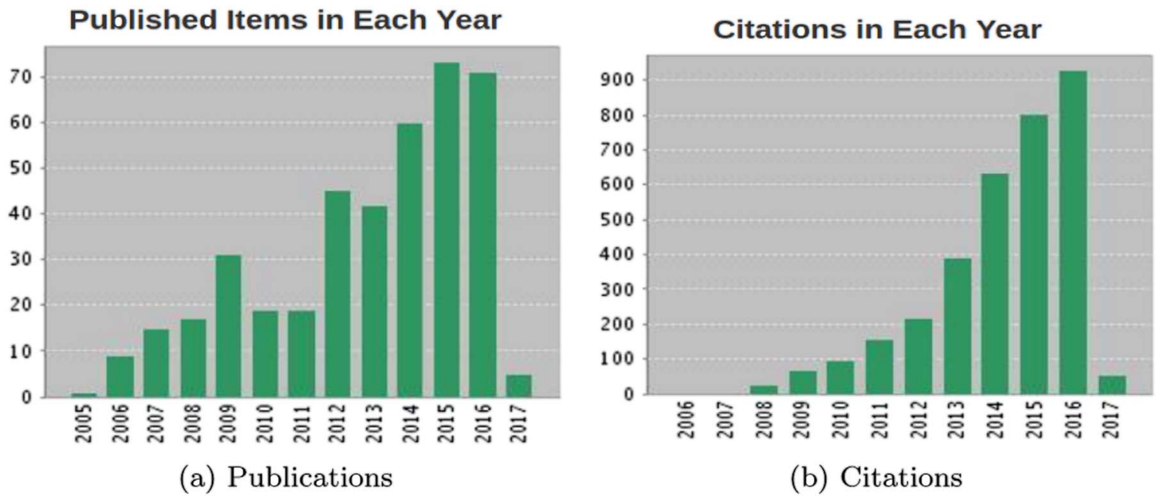


Fig. 1. The number of publications and citations by year.

and image processing, identification problem, bioinformatics, and medicine, etc.

The remainder of this paper is organized as follows: Section 2 is described the basic concepts of OBL. Section 3 is theoretical research works on opposition schemes. Developmental research works conducted on OBL is described in Section 4. In Section 5, the applications of OBL is presented. Finally, the paper is concluded in Section 6.

2. Opposition-based learning: basic concepts and pioneering research works

In this section, first we summarize the basic concepts of OBL. Then, pioneering research works on using OBL concept in the machine learning algorithms such as EA, RL, ANN, and Fuzzy Systems are explained.

2.1. Basic concepts

The primary opposition concept first was expressed in the Yin-Yang symbol (Fig. 2) in the ancient Chinese philosophy [205]. This symbol indicates the duality concept in which black and white are Yin (receptive, feminine, dark, passive force) and Yang (creative, masculine, bright, active force), respectively. Also, Greek classical elements of nature patterns (Fig. 3) described the opposition concepts such as fire (hot and dry) vs. water (cold and wet), earth (cold and dry) vs. air (hot and wet). Cold, hot, wet, and dry indicate nature entities and their opposite entities [205]. It seems that the concept of many entities or situations in the real-world is described by using the opposition concept. In fact, using the opposition concept makes the explanation of different entities much easier. Pair-wised opposites such as the east, west, south, and north cannot be defined alone and only they can explain in terms of one another. Therefore, the computational

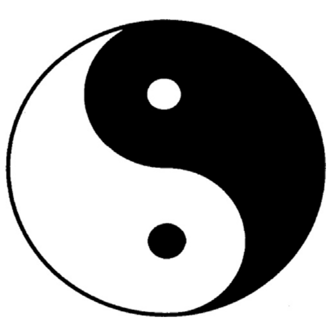


Fig. 2. Early opposite concept was mentioned in the Yin-Yang symbol [205].

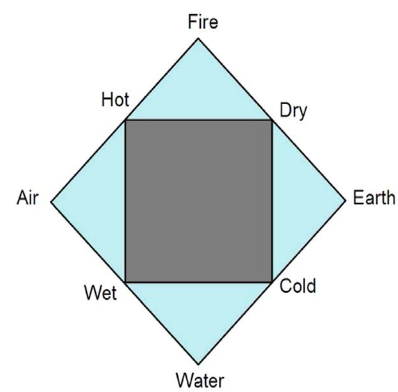


Fig. 3. The Greek classical elements to explain patterns in the nature [205].

opposition concept [297] was inspired from the opposition concept in the real-world and the opposite numbers were simply defined in [297] as follows.

Definition 1 (*Opposite number***).** [297] Let $x \in [a, b]$ be a real number. Its opposite, $\breve{x}$, is defined as follow:

$$\breve{x} = a + b - x, \tag{1}$$

The extended definition for the higher dimension is defined in [296,297] as follows.

Definition 2 (*Opposite point in the D space***).** [297] Let $x(x_1, ..., x_D)$ be a point in D-dimensional space and $x_i \in [a_i, b_i]$, $i = 1, 2, ..., D$. The opposite of x is defined by $\breve{x}(\breve{x}_1, ..., \breve{x}_D)$ as follow:

$$\breve{x}_i = a_i + b_i - x_i \tag{2}$$

In fact, they indicate that for finding the unknown optimal solution, searching both a random direction and its opposite simultaneously gives a higher chance to find the promising regions. It is reasonable that if the current estimates (guesses) are far away from the unknown optimal solution, computing their opposites leads to the opposite direction toward to the unknown optimal solution. Note that the basic opposite point is computed same as a reflected point when it is calculated through the center point $((x_1 + x_2 + ... + x_D)/2)$ [1].

The above definition of the opposite point is called as Type-I opposite. The Type-I opposite is defined according to the relationship between points in the search space without considering their objective values. Fig. 4 indicates x and its opposite, $\breve{x}$, in one, two, and three-dimensional spaces.

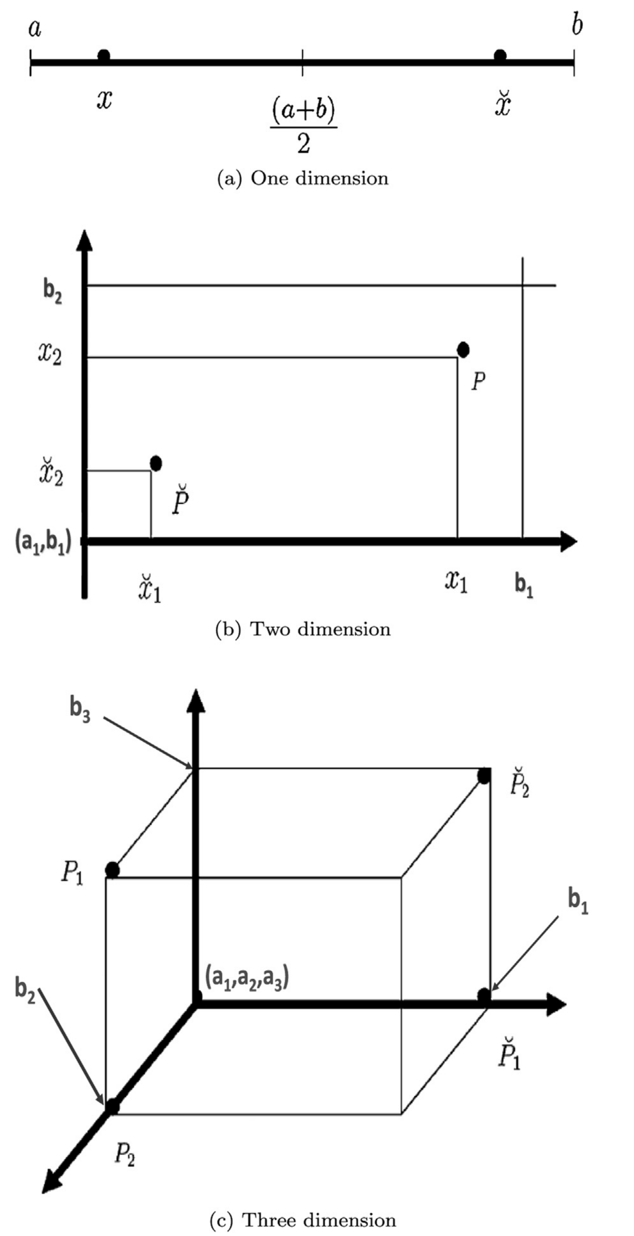


Fig. 4. The point x and its corresponding Type-I opposite in one, two, and three-dimensional spaces [210] in the interval $[a_i, b_i]$.

In [296], the Type-II opposition was defined according to the objective space of a problem as follows.

Definition 3 (*Type-II Opposite Points*). [296] Suppose that for the function $f(x_1, x_2, \ldots, x_D)$, y_{min} and y_{max} are predefined or can be estimated. Let $y = f(x_1, x_2, \ldots, x_D) \in R$ be an arbitrary function with $y \in [y_{min}, y_{max}]$. For every point $x(x_1, \ldots, x_D)$, the Type-II opposite point $\breve{x}(\breve{x}_1, \ldots, \breve{x}_D)$ is defined as follow:

$$\breve{x}_{II} = \{x | \breve{y} = y_{min} + y_{max} - y\} \tag{3}$$

Fig. 5 indicates Type-I and Type-II opposite points for a sample function landscape.

There are many research works which have employed the OBL concept to enhance the performance of searching, learning, or optimization algorithms. We categorize all these research works into three main areas: theoretical, developmental, application oriented research works. The following sections provide a brief overview of the OBL-based algorithms and the new concept and description of OBL. In the following sections, we simply mention Type-I opposition point as the opposite point. Type-II opposition requires a priori knowledge of the objective space so in black-box problems, it is very complicated to compute Type-II opposition. Fig. 6 presents the major types of Type-I opposition which can be utilized for Type-II opposition as well; and the following subsections provide a brief overview of them.

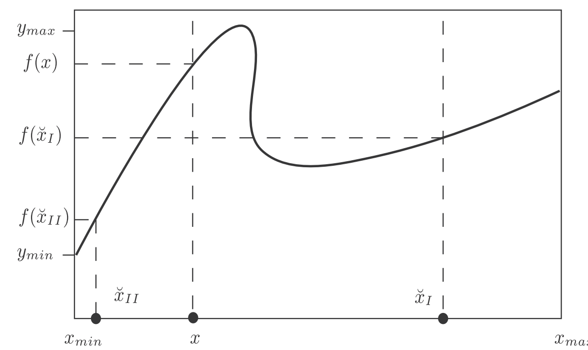


Fig. 5. Definition of Type-I versus Type-II opposition for a sample landscape [296].

2.2. Pioneering research works

The first effort of utilizing the OBL concept in an optimization method was proposed by Rahnamayan et al. in 2006 [208]. As a case study, OBL was used in the DE algorithm to improve its performance, the resulted algorithm is well-known as the Opposition-based DE (ODE). In the proposed scheme, OBL is applied in two stages of the classical DE algorithm; during initialization of population and evolutionary process. In the initialization step, the initial population is randomly generated (line 3) and simultaneously the opposite population is calculated by computing the opposite of each candidate solution in the population (line 4). Then, two populations, the initial population and its corresponding opposite, are combined and the fittest solutions are selected as the initial population (line 5). During the evolutionary process, a jumping rate (i.e., jumping probability) is defined (line 12) which based on the jumping rate, the opposite of each variable in the candidate solution is computed dynamically. First, the minimum and maximum values of each variable in the current population are utilized to calculate the opposite of the current population (line 14). Then, both the current and its opposite are combined and the fitter solutions are selected (line 16) as the mentioned approach in the initialization phase. Dynamic opposite for the candidate solution x in the evolutionary process is calculated as follow:

$$\breve{x}_{i,j} = a_i + b_i - x_{i,j}, \tag{4}$$

where a_i and b_i are maximum and minimum values of each variable in the current population. In addition, opposition-based population jumping is applied based on a predefined jumping rate, J_r. Algorithm 1 presents all main steps of the ODE algorithm.

Algorithm 1. ODE scheme (NP, J_r, $MAXNFC$).

1: //NP, J_r, and $MAXNFC$ are the population size, jumping rate, and the maximum number of function evaluations, respectively.
2: **// Opposition-Based Population Initialization**;
3: Generating the initial population uniform randomly, *pop*;
4: Calculating the opposite population, *opop* by using Eq. (2);
5: Picking NP fittest solutions from $pop \bigcup opop$ as the initial population;
6: NFC=1;
7: **while** NFC < MAX_NFC
8: Mutation;
9: Crossover;
10: Selection;

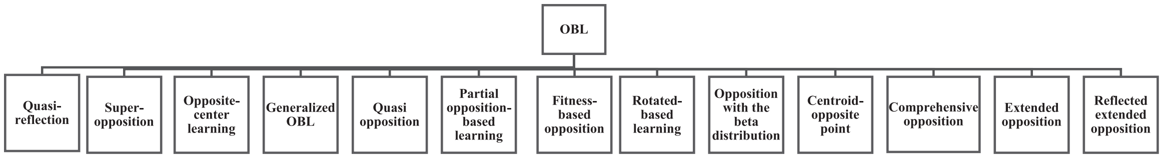


Fig. 6. The major variants of Type-I opposition which similarly can be extended to the Type-II opposition.

// **Opposition-Based Jumping**;
if *rand* (0, 1) < J_r **then**
  // rand(0,1) generates a random number in [0, 1];
  Calculating opposite population of current population, *opop*(*NFC*) by Eq. (4);
  $NFC = NFC + NP$;
 Picking *NP* fittest solutions from $pop \bigcup opop$ as the current population;
end if
 $NFC = NFC + NP$;
end while

In [210], the comprehensive experiments were performed to verify the performance of ODE by using a benchmark function set including 58 different global optimization problems. The effect of dimensionality, opposite points, population size, different strategies of mutation and jumping rates were investigated and analyzed. The achieved results indicated that ODE obtains superior results than DE for high-dimensional problems and also, ODE performs better when the population size is increased. In addition, for the jumping rate, the range of [0.1, 0.4] was suggested for a black-box optimization problem.

In 2005, the first attempt of enhancing RL method was made by Tizhoosh [298] by using the OBL concept. OBL is extended to the action a in the Q-learning algorithm. For finding opposite actions, a degree of opposition $\breve{Q}$ for two actions a_1 in the state s_i and a_2 in the state s_j are defined as follows:

$$\breve{Q}(a_1|s_i, a_2|s_j) = \eta(s_i, s_j) \times \left[1 - exp\left(-\frac{Q(s_i, a_1) - Q(s_j, a_2)}{max_k(Q(s_i, a_k), Q(s_j, a_k))}\right)\right], \tag{5}$$

$$\eta(s_i, s_j) = 1 - \frac{\sum_k |Q(s_i, a_k) - Q(s_j, a_k)|}{\sum_k max(Q(s_i, a_k), Q(s_j, a_k))}, \tag{6}$$

After determining the opposite action $\breve{a}$, the opposite reward $\breve{r}$ is also calculated. An opposite learning step $\breve{\alpha}$ is defined as $\sqrt{1 - \frac{i}{n_E}}$ for the iteration i; n_E is the total number of iterations. Q-value of the opposite action $\breve{a}$ is computed as follow:

$$Q(s, \breve{a}) = Q(s, \breve{a}) + \breve{\alpha}[\breve{r} + \gamma \max_{a'}(Q(s', a')) - Q(s, \breve{a})] \tag{7}$$

It can be seen the difference of Q-matrix updating with n actions and m states between the RL and opposition-based RL from Fig. 7 for the action a_2 in the state s_2.

Also, the first effort in using the OBL concept for backpropagation ANN was proposed in [307]. The opposite transfer function of $f(x)$ is defined by multiplying all weights of neurons by −1; i.e., $f(-x)$. The opposite network contains the same weights with the original network, but at least one neuron is utilizing the opposite transfer function. Fig. 8 shows the hyperbolic tangent function and its opposite.

The first effort of defining the opposite fuzzy sets was proposed by Tizhoosh [299]. For the description of the opposite fuzzy sets, a fuzzy set $A \subset X$ with membership function $\mu_A(x)$ is given by:

$$A = \{(x, \mu_A(x))|x \in X, \mu(x) \in [0, 1]\}, \mu_A(x) = f(x, a, \delta), \tag{8}$$

where $\mu_A(x)$ is equal to one for $\forall\ a_i \in a$. Then, the opposite of a fuzzy set is described as follows.

Definition 4 (*Opposite fuzzy set*). [299] The opposite of a fuzzy set $A \subset X$ is defined as:

$$\breve{A} = \{(x, \mu_{\breve{A}}(x))|x \in X, \mu_{\breve{A}}(x) \in [0, 1]\}, \tag{9}$$

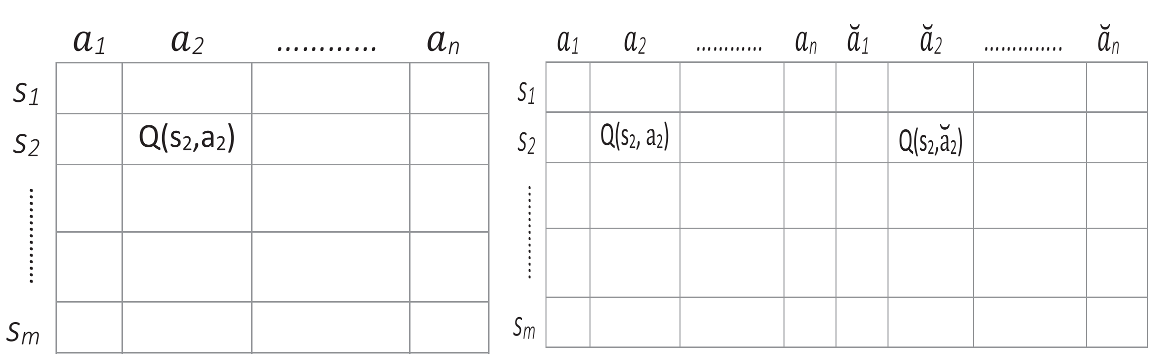
where $\mu_{\breve{A}}(x) = f(x, \breve{a}, \breve{\delta})$ and the vector a and the vector $\breve{a}$ are the points on the universe of discourse such that $\mu(a) = \mu(\breve{a}) = 1$. Also, other schemes of opposite points such as super, quasi, and Type-II opposite points are described as follows.

Definition 5 (*Super opposite fuzzy set*). [299] The super opposite of a fuzzy set, $\breve{A}^S$, is defined with the spacial membership function, $\mu_{\breve{A}^S}(x) = f(x, \breve{a}^S, \breve{\delta}^S)$, in which $\breve{a}^S$ and $\breve{\delta}^S$ are the super opposite of a and δ in the fuzzy set A with the membership function $\mu_A(x) = f(x, a, \delta)$.

Definition 6 (*Quasi opposite fuzzy set*). [299] Quasi opposite fuzzy of a set, $\breve{A}^q$, is defined with the spacial membership function, $\mu_{\breve{A}^q}(x) = f(x, \breve{a}^q, \breve{\delta}^q)$, in which $\breve{a}^q$ and $\breve{\delta}^q$ are the quasi opposite of a and δ in the fuzzy set A with the membership function $\mu_A(x) = f(x, a, \delta)$.

Definition 7 (*Type II opposite fuzzy set*). [299] Type II opposite of a fuzzy set, $\breve{A}_{II}$, is defined with the spacial membership function, $\mu_{\breve{A}_{II}}(x) = f(x, \breve{a}_{II}, \breve{\delta}_{II})$, in which $\breve{a}_{II}$ and $\breve{\delta}_{II}$ are the type II opposite of a and δ in the fuzzy set A with the membership function $\mu_A(x) = f(x, a, \delta)$.

A new image segmentation algorithm was introduced according to the defined opposite fuzzy sets in [299]. In this method, first the set A is



(a) Q-matrix for RL (b) Q-matrix for opposition-based RL

Fig. 7. Q-matrix updating.

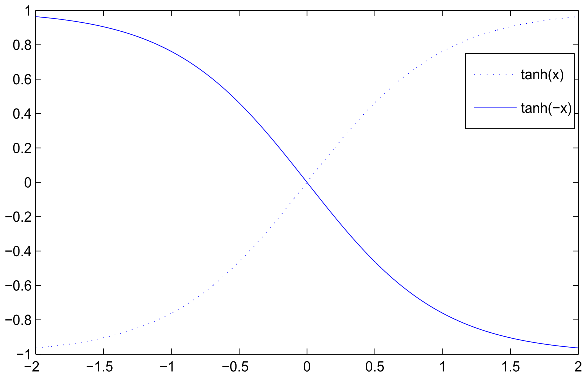


Fig. 8. Hyperbolic tangent function $tanh(x)$ and its opposite, $tanh(-x)$.

defined as dark pixels in the image. Then, the opposite fuzzy sets are calculated iteratively with different sizes, starting from the brightest region. The differences among the entropy of a fuzzy set and its opposite sets are computed and the set with the minimum entropy difference is selected as the opposite set for the set A.

3. Theoretical research works on opposition schemes

This section briefly describe some research works which focus on defining new opposition types or mathematical theorems to indicate their usefulness. After the first efforts of using the OBL concept in the machine learning algorithms, the new opposition-based algorithms are being extended rapidly. In the following subsection, the new definition of opposition schemes and corresponding theorems are briefly described. These research works are summarized as in Table 1. Also, the section is organized into three main subsections including research works on EA, RL, and ANN.

3.1. EA-related research works

Several research works have been conducted in utilizing OBL concept and extending the new schemes of OBL to enhance EA algorithms. Also, mathematical theorems were derived to demonstrate that opposite candidate solutions has higher probability to be closer to an unknown optimal solution than the randomly generated candidate solutions.

3.1.1. Theorems related to OBL

In [210,216], some theorems are mathematically proved to conclude the advantage of using the OBL concept. Also, they conducted some experiments by utilizing the OBL concept in the framework of EA algorithms to enhance their performance.

The following theorem indicates that the opposition of a point is unique.

Theorem 1 (*Uniqueness*). [210] *In the D dimensional space, for each point* $x(x_1, \ldots, x_D)$ $(x_i \in [a_i, b_i])$, *there is a unique opposite point* $\check{x}(\check{x}_1, \ldots, \check{x}_D)$ *according to the mentioned definition* $\check{x}_i = a_i + b_i - x_i$, $i = 1, 2, \ldots, D$.

The following theorem indicates the closeness probability of a candidate solution and its opposite to the unknown optimal solution is equal.

Theorem 2. [210] *Suppose that* $f(x)$ *is an unknown function with the optimal solution* $x_s (x_s \neq (a+b)/2)$ *and* x *is a candidate solution and its opposite is* $\check{x}$. *Then,*

$$Pr(|\check{x} - x_s| < |x - x_s|) = \frac{1}{2}, \tag{10}$$

where $Pr()$ *is the probability function.*

The below theorem shows that by assuming an objective function is a monotone, the opposite candidate solution $\check{x}$ has a higher chance (12.5%) to be closer to an unknown optimal solution compared to the second random candidate solution x_r.

Theorem 3 (*Second opposition*). [210] *Suppose that* $g(x)$ *is increasingly monotone function and* x, $\check{x}$, *and* x_r *are the first random candidate solution, the opposite point of* x; *and the second random candidate solution, respectively. Then,*

$$Pr(g(x) < max\{g(x), g(\check{x})\}) = \frac{3}{4}, \tag{11}$$

The following theorem is derived to indicate that without any assumption, the opposite candidate solution $\check{x}$ has higher probability to

Table 1
A summary of major research works on definitions of opposition schemes and mathematical proofs of opposition.

Author	Brief explanation of research work with theoretical contribution
Rahnamayan et al. [210]	Having a higher chance by utilizing random numbers and their opposites mathematically was proved.
Rahnamayan et al. [216]	Some theorems and definitions were extended to intuitively indicate that utilizing the opposite of a candidate solution is beneficial.
Rahnamayan et al. [209]	Quasi-oppositional DE (QODE) was proposed as a uniform random point between the center point and the opposite point.
Tizhoosh et al. [302]	Super-opposition scheme and Type-II opposition concept were introduced.
Ergezer et al. [77]	Fitness-based opposition (FBO) and quasi-based reflection were introduced based on the concept of the quasi opposite point.
Wang et al. [329]	They introduced three different schemes of the generalized OBL (GOBL).
Tang and Zhao [286]	The modified type of OBL was proposed by conducting opposition on multiple points.
Ao [12]	The modified OBL concept was proposed as a second mutation for DE algorithm.
Kushida et al. [133]	An archived OBL was introduced based on the archive best solutions.
Hu et al. [105]	A partial opposition-based learning (POBL) scheme was proposed.
Ergezer et al. [75]	The mathematical theorems were proved to analyze the effect of using three OBL types (opposition, quasi opposition and quasi-reflection).
Liu et al. [148]	Rotated-Based Learning (RBL) was proposed by rotating any specified deflection angle.
Rahnamayan et al. [206]	The centroid-opposite was proposed according to the gravity center of population.
Seif and Ahmadi [245]	The mathematical theorems and proofs of the opposition concept were introduced for binary optimization problems.
Seif and Ahmadi [245]	Comprehensive opposition (CO) was introduced by using of Quasi-opposition and reflection and defining two new concepts of extended opposition (EO) and reflected extended opposition (REO).
Park and Lee [192]	A stochastic OBL using a beta distribution with the selection switching and the partial dimensional changing schemes was proposed.
Ergezer and Simon [76]	The fitness-based quasi-reflection concept (FQR) was introduced.
Rahnamayan et al. [214,213,215]	The concept of center-based sampling was proposed and indicated that the probability of closeness for the center point/region to an unknown solution is higher than other points.
Xu et al. [355]	Opposite-Center Learning (OCL) was proposed based on minimizing the distance of the pair including the original candidate and the opposite point to the global optimum.
Shokri et al. [263]	Oppositional target domain estimation (OTE) was proposed to reduce the search and navigation area.
Dhahri et al. [59]	The OBL concept was modified and applied to DE algorithm for the design of a beta basis function neural network (BBF).
Ventresca and Tizhoosh [311]	Mathematical theorems and proofs were developed to investigate the effectiveness of employing opposite transfer functions.

be close to an unknown optimal solution compared to the second random candidate solution x_r.

Theorem 4 (*Central opposition*). [210] *Suppose that $f(x)$ is an unknown function with $x(x_1,\ldots,x_D)$, $x_i \in [a_i, b_i]$, $i = 1, 2, \ldots, D$ and at least one solution $x_s(x_{s1},\ldots,x_{sn})$ and $x(x_1,\ldots,x_D)$, $\breve{x}(\breve{x}_1,\ldots,\breve{x}_D)$, and x_r are the first random candidate solution, the opposite point of x; and the second random candidate solution. Then,*

$$Pr(d(\breve{x}, x_s) < d(x_r, x_s)) > Pr(d(x_r, x_s) < d(\breve{x}, x_s)), \tag{12}$$

where $d(.)$ is the Euclidean distance.

The mathematical proofs in [210] have two shortcomings: (1) they established only for one dimensional space, (2) they don't provide an intuitive interpretation. In [216], some simpler mathematical proofs were given to intuitively indicate the benefit of using opposition concept according to distance to the optimal solution in the high dimensional search space. Some theorems and definitions are described to show that the distance to an unknown optimal solution by the opposition-based sampling is less than the random sampling according to the Euclidean distance. The expected distance between two random points $x, y \in [a, b]$ against x and $\breve{x}$ is computed as follow:

$$E[|x - y|] = L/3,\ L = b - a,\ E[|x - \breve{x}|] = L/2 \tag{13}$$

They calculated the three following probabilities, the probability of the candidate solution x, its opposite $\breve{x}$, and the second random candidate solution r being closer to some unknown optimal solution $s \in S$, described by:

$$P_x = Pr(d(x, s) < d(\breve{x}, s) \wedge d(x, s) < d(r, s)), \tag{14}$$

$$P_{\breve{x}} = Pr(d(\breve{x}, s) < d(x, s) \wedge d(\breve{x}, s) < d(r, s)), \tag{15}$$

$$P_{rand} = Pr(d(r, s) < d(\breve{x}, s) \wedge d(r, s) < d(x, s)), \tag{16}$$

First, in one, two, and three dimensional space, the probabilities of P_x, $P_{\breve{x}}$, and P_{rand} are calculated as 0.375, 0.375, and 0.25, respectively. Then, these probabilities are mathematically derived for N dimensional search space by the following theorem.

Theorem 5 indicates that considering the random candidate solution and its opposite is more likely compared to the paired random candidate solutions according to Euclidean distance in an N-dimensional hypercube search space.

Theorem 5 (*Central opposition*). [216]*Suppose that H_0^D be an D dimensional hypercube with edge lengths $L_0 > 0$ and H_i^D be an inscribed hypercube having a common center to H_0^D with following edge lengths:*

$$L_i(D) = \begin{cases} \dfrac{L_0}{4}, & \text{if} \quad D = 1 \\ \frac{L_0\sqrt{2}}{4}, & \text{if} \quad D \leq 1 \end{cases} \tag{17}$$

Then, the volume of H_i^D and P_{rand} will be $1/(2D + 1)$ and 0.25, respectively.

Also, some examples of symmetric and non-symmetric evaluation functions are analyzed to confirm the proof of the above theorem.

3.1.2. Quasi appositions and their corresponding theorems

In [209], the quasi opposite of the point x was introduced as a uniform random point generated between center point and the opposite point $\breve{x}$. Fig. 9 shows x, $\breve{x}$, and the quasi opposite point (x^q) in one, two, and three-dimensional spaces.

They prove mathematically in the following theorem that the probability of the quasi opposite point x^q being closer to the unknown optimal solution is higher than the opposite point x.

Theorem 6. *Assume that x_r, x_o, and x^q are a random uniform candidate solution, its opposite, and its quasi opposite, respectively, and x_r is the unknown optimal solution. Then*

$$Pr[d(\breve{x}^q, x_s) < d(\breve{x}, x_s)] > 1/2, \tag{18}$$

where $d(.)$ is the distance from x_s, optimal solution. This theorem is proved for one-dimensional space, but in the same way it can be proved for the higher dimension space.

Two new definitions, fitness-based opposition (FBO) and quasi-based reflection, were introduced by using the concept of the quasi opposite point [77]. The quasi-reflected point x_{q_r} for the candidate solution x is defined as a random point uniformly distributed between $c = (a + b)/2$ and x. Fig. 10 shows $\breve{x}$, x_q and x_{q_r} in one-dimensional space.

FBO is defined by embedding a reflection weight in a generated quasi-reflected solution between the current candidate solution and the median of population to determine the amount of reflection based on the solution fitness value. For a candidate solution x, its FBO, x_{fbr}, is defined as follow:

$$x_{fbr} = \begin{cases} x + (median - x)k & \text{if} \quad x < median \\ median + (x - median)k & otherwise \end{cases}, \tag{19}$$

which $k \in [0, 1]$ is a uniform random number. Furthermore, they indicate that the quasi-opposite point is usually closer than an opposite point to the optimal solution in one-dimensional search space by computing the corresponding probability. Also, they analyzed the performance of quasi-reflection and quasi-opposite through simulations and demonstrated that their performance is improved with increasing the dimension of problems. In addition, the new concepts are applied to BBO algorithm to increase its performance. In [75], the mathematical proofs were provided to analyze the effect of using three opposition types (opposition, quasi opposition and quasi-reflection) in EA algorithms. They illustrated that the closeness probability of the different types of opposite points to an unknown solution is more than a random candidate solution. In the $1D$ search space, the closeness probability of the different types of opposite points to an unknown solution x_{opt} compared to a random candidate solution x is derived as follow:

$$Pr[|\breve{x}_q - x_{opt}| < |x - x_{opt}|] = \frac{9}{16}, \quad Pr[|\breve{x}_{q_r} - x_{opt}| < |x - x_{opt}|] = \frac{11}{16}, \tag{20}$$

where $\breve{x}_{qo}$ and $\breve{x}_{q_r}$ are its quasi-opposite and quasi-reflected, respectively. Also, the closeness probability of the quasi-reflected and quasi-opposition to unknown solution x_{opt} are higher than its opposite:

$$Pr[|\breve{x}_q - x_{opt}| < |\breve{x}_o - x_{opt}|] = \frac{11}{16}, \quad Pr[|\breve{x}_{q_r} - x_{opt}| < |\breve{x}_o - x_{opt}|] = \frac{9}{16}, \tag{21}$$

$\breve{x}_o$ is the opposite of the point x. They indicate that the quasi-reflection has higher closeness probability compared to other OBL types. They compared the performance of oppositional-based learning on GA, DE, and BBO algorithms. In [76], the fitness-based quasi-reflection concept (FQR) was introduced which controls the amount of reflection based on the fitness of the individual. FQR for a candidate solution x is given by:

$$x_{fqr} = \begin{cases} x + (c - x)k & \text{if} \quad x \leq c \\ c + (x - c)(1 - k) & \text{if} \quad x > c \end{cases} \tag{22}$$

where c is the center point and k is the rank of solution based on its fitness in the population. Different reflection types of k is designed by using various complementary functions for k. They provide the mathematical proofs to compute the closeness probability of x_{fqr} to an unknown solution. This probability for the $1D$ space is defined as a function of the reflection weight k; i.e., $(6 - k)/8$. Using simulation, they indicated the closeness probability of x_{fqr} is increased on higher dimensions.

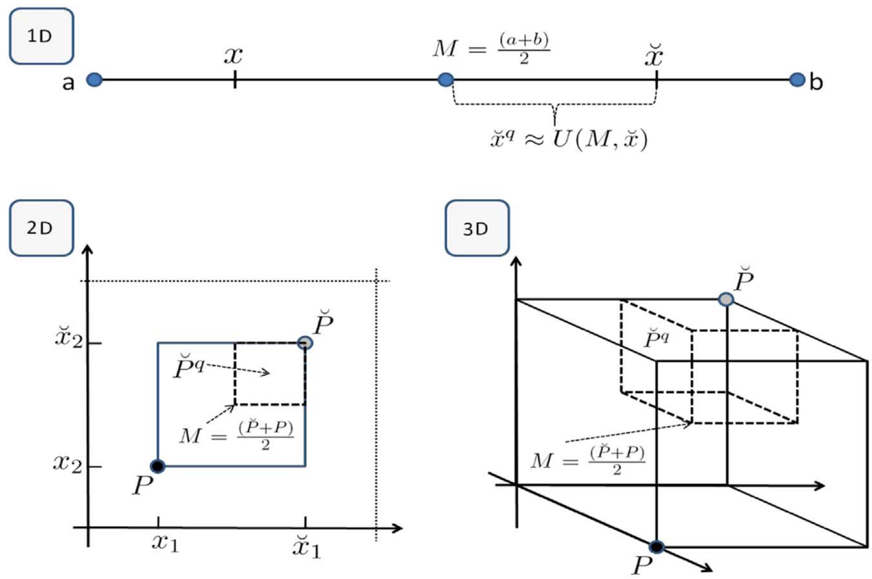


Fig. 9. Opposite-point and quasi-opposite region (marked by dashed lines) for 1D, 2D, and 3D search spaces [214].

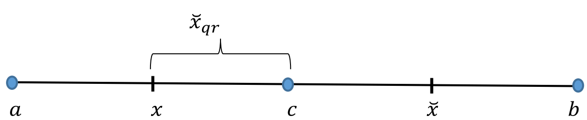


Fig. 10. Opposite-point and quasi-reflected region x_{q_r}.

3.1.3. Schemes for binary search space and their corresponding theorems

Seif et al. [245] introduced the mathematical theorems and proofs of the opposition concept for binary optimization problems. For binary spaces, the Hamming distance is utilized. The opposite point $\breve{x}$ for a binary point $x(x_1, x_2, \ldots, x_D), \quad x_i \in \{0, 1\}$ is defined as:

$$\forall\ X(x_1, x_2, \ldots, x_D) \text{ with } x_i \in \{0, 1\}, \text{ for each } \breve{x}_i = 1 - x_i \text{ in } \breve{X}(\breve{x}_1, \breve{x}_2, \ldots, \breve{x}_D), \tag{23}$$

They proved as a theorem that every point $x, \quad x \in \{0, 1\}$ has a unique opposite point based on the above definition.

Also, in the following theorem they prove that the opposite points have a higher chance to be closer to an optimal solution based on the Hamming distance.

Theorem 7. *Assume that* $x(x_1, x_2, \ldots, x_D), \quad x_i \in \{0, 1\}$ *and* $x_r(x_{r1}, x_{r2}, \ldots, x_{rD}), \quad x_{ri} \in \{0, 1\}$ *be the first and second random candidate solutions in the binary solution space, respectively. Then*

$$Pr(\min\{\| x, x_s \|, \| \breve{x}, x_s \|\} \le \min\{\| x, x_s \|, \| x_r, x_s \|\}) > \tag{24}$$

$$Pr(\min\{\| x, x_s \|, \| x_r, x_s \|\} \le \min\{\| x, x_s \|, \| \breve{x}, x_s \|\}), \tag{25}$$

where x_s is the optimal solution.

This theorem describes briefly that the candidate solution x and its opposite $\breve{x}$ are more likely to be closer to the optimal solution than the first and second random candidate solutions, x and x_r. Furthermore, they utilize OBL during population initialization and also during the evolutionary process to enhance the performance of binary GSO algorithm.

3.1.4. Comprehensive opposition and its corresponding theorems

A new OBL concept, Comprehensive Opposition (CO), was introduced in [244] by using of Quasi-opposition and reflection and two new concepts of Extended Opposition (EO) and Reflected Extended Opposition (REO). In following, corresponding definitions and theorems are briefly described. Let x be a solution in n-dimensional space, its EO, $\breve{x}_i^{eo}$, and REO, $\breve{x}_i^{reo}$, are defined as follows:

$$\breve{x}_i^{eo} = \begin{cases} rand(\breve{x}_i, b_i) & x_i < (a_i + b_i)/2 \\ rand(a_i, \breve{x}_i) & x_i > (a_i + b_i)/2, \end{cases} \quad \text{for } i = 1 \ldots n, \tag{26}$$

$$\breve{x}_i^{reo} = \begin{cases} rand(x_i, b_i) & x_i > (a_i + b_i)/2 \\ rand(a_i, x_i) & x_i < (a_i + b_i)/2, \end{cases} \quad \text{for } i = 1 \ldots n, \tag{27}$$

The following theorems 8, 9, and 10 indicate that the mentioned opposition schemes generate points closer to a solution than a random candidate solution, according to the expected distance and values of the closeness probability. They proved the following theorem which computes the expected values of the probability for the quasi-opposite ($\breve{x}_{qo}$), quasi-reflected opposite ($\breve{x}_{qr}$), extended opposite ($\breve{x}_{eo}$), and reflected extended opposite ($\breve{x}_{reo}$) being closer than a random candidate solution to an unknown optimal solution, x^*.

Theorem 8. [244] *Suppose x be a random candidate solution, then the expected values of the probability of $\breve{x}_{qo}$, $\breve{x}_{qr}$, $\breve{x}_{eo}$, and $\breve{x}_{reo}$ being closer than a random candidate solution to x^* are as follows:*

$$E(Pr(|\breve{x}_{reo} - x^*| < |x - x^*|)) = 3/16, \tag{28}$$

$$E(Pr(|\breve{x}_{qr} - x^*| < |x - x^*|)) = 11/16, \tag{29}$$

$$E(Pr(|\breve{x}_{qo} - x^*| < |x - x^*|)) = 9/16, \tag{30}$$

$$E(Pr(|\breve{x}_{eo} - x^*| < |x - x^*|)) = 7/16, \tag{31}$$

Also, the expected distance of a random candidate solution x and its variant opposites are calculated in the following theorem.

Theorem 9. [244] *Assume that x has the uniform distribution in $[a, b]$. Then, the expected distance x and its various opposites are calculated as follows:*

$$E(|x - \breve{x}_{reo}|) = (b - a)/8, \tag{32}$$

$$E(|x - \breve{x}_{qr}|) = (b - a)/8, \tag{33}$$

$$E(|x - \breve{x}_{qo}|) = 3(b - a)/8, \tag{34}$$

$$E(|x - \breve{x}_{eo}|) = 5(b - a)/8, \tag{35}$$

After defining $\breve{x}^{eo}$ and $\breve{x}^{reo}$ and their corresponding theorems, CO is defined to improve the candidate solution and control the diversity of the population. Let the probabilities of selected opposite points $\breve{x}_{qo}$, $\breve{x}_{qr}$, $\breve{x}_{eo}$, and $\breve{x}_{reo}$ for the candidate solution x be P_{reo}, P_{qr}, P_{qo}, and P_{eo}, respectively. Then, CO is defined as follows:

$$\breve{x}_{co} = \begin{cases} \breve{x}_{reo}, & rand \le P_{reo} \\ \breve{x}_{qr}, & P_{reo} < rand \le P_{reo} + P_{qr} \\ \breve{x}_{qo}, & P_{reo} + P_{qr} < rand \le P_{reo} + P_{qr} + P_{qo} \\ \breve{x}_{eo}, & P_{reo} + P_{qr} + P_{qo} < rand \le 1 \end{cases} \tag{36}$$

Also, they considered the problem of finding the optimum value of P_{reo}, P_{qr}, P_{qo}, and P_{eo} and suggested some specific values for them. The following theorem is derived to show that why a comprehensive opposite point is more effective than an independent random point.

Theorem 10. [244] *Assume that x_r is the second random uniform solution. For a random candidate solution x and its comprehensive opposite $\breve{x}_{co}$, then:*

$$0.505 < E\left(Pr\left(|\breve{x}_{co} - x^*| < |x - x^*|\right)\right) \le 0.67875, \tag{37}$$

$$E\left(Pr\left(|x_r - x^*| < |x - x^*|\right)\right) = 0.5, \tag{38}$$

$$E\left(Pr\left(|\breve{x}_{co} - x| > |x_r - x|\right)\right) \quad \text{while} \quad t/T < 11/36, \tag{39}$$

where t is the iteration number and T is the total number of iterations.

In addition, a Markov chain model of the opposition-based metaheuristic optimization algorithm (OBA) is described to indicate that with probability one, OBA converges to a global optimum. Finally, experiments were conducted based on OBA for enhancing PSO and GSA algorithms.

3.1.5. Other OBL schemes

In [302], super-opposite points were defined as all points $\breve{x}^s$ where $d(\breve{x}^s, x) > d(\breve{x}, x)$, where $d(.,.)$ is a metric such as Euclidean distance. Then, the super-opposite relationship for the point x is defined by:

$$\breve{x}^s \in \begin{cases} [a, \breve{x}) & x > \dfrac{(a+b)}{2} \\ [a, b] - \{x\} & x = \dfrac{(a+b)}{2} \\ [\breve{x}, b] & x < \dfrac{(a+b)}{2} \end{cases} \tag{40}$$

The first version of the generalized OBL (GOBL) was defined for the candidate solution x in $[a, b]$ in [328] as $\breve{x} = \Delta - x$ where Δ is a computable value and $\breve{x}$ is in $[\Delta - b, \Delta - a]$. In [329], the new definition of GOBL is described as follow:

$$\breve{x} = k(a + b) - x, \tag{41}$$

where k is a real number. Three different schemes of GOBL are defined based on the value k as follow:

1. Symmetrical opposite candidate solutions in GOBL (GOBL-SS) are generated by $k = 0$, $\breve{x} = -x$.
2. A symmetry of opposite interval (GOBL-SI) is defined by $k = \frac{1}{2}$, $\breve{x} = \frac{a+b}{2} - x$.
3. Random GOBL (GOBL-R) is defined as $\breve{x} = k(a + b) - x$, where k is a uniform random number in the interval $[0, 1]$.

Based on the definition of GOBL, it is possible to violate the box-constraints, $x_i \notin [a_i, b_i]$, a random number is generated within $[a_i, b_i]$ as GOBL value of x_i.

In [286], the modified type of OBL was proposed by conducting opposition on multiple points in which two points are recombined and then its opposite is computed. For a candidate solution P_i, another solution P_{i1} is randomly generated and the opposite of combined candidate solution is computed as follows:

$$OP'_i = a + b - (m_1 \cdot P_i + m_2 \cdot P_{i1}), \tag{42}$$

where m_1 and m_2 are two uniform random numbers in $[0, 1]$, and $m_1 + m_2 = 1$. The modified OBL concept was defined as a second mutation for DE algorithm in [12]. The modified version of OBL is defined as follow:

$$O_{ij} = \begin{cases} a_j + b_j - x_{ij}, & \text{if} \quad j \in S \subseteq \{1, 2, \ldots, n\} \\ x_{ij}, & \text{otherwise} \end{cases} \tag{43}$$

After DE algorithm generates the offspring population, the modified version of OBL is applied in the half worst of offspring population. An OBL was defined in [358,377] based on the current best candidate solution, x_{best}, for the candidate solution x as follows:

$$2x_{best} - x \tag{44}$$

It is applied to the population initialization and during the evolution process of DE. In [133], an archived OBL was introduced based on the archive best solutions. In this method, first the difference between the current best candidate solution 'xbest' and the selected random candidate solution is calculated as $\sigma = x_{best} - x_a$. The archived opposition for point x_i is computed as follows:

$$M_i(j) = (a_i(j) + b_i(j))/2,$$
$$x_i(j) = 2\{k \cdot M_i(j) + (1 - k) \cdot (M_i(j) + \sigma_j)\} - x_i(j), \tag{45}$$

where $a_i(j)$ and $b_i(j)$ are the minimum and maximum values for the dimension j of the population. A partial opposition-based learning (POBL) scheme was proposed in [105] which the opposition of some dimensions is computed according to a random uniform probability; a random value between 0.1 and 0.3. In this method, the original point, the opposite point, and partial opposite point are compared and the better one is selected. In [148], Rotation-Based Learning (RBL) was proposed by rotating any specified deflection angle. The opposition of a point can be defined as 180 degrees rotation of the original point in $2D$ space so this concept is extended to compute the rotation with any arbitrary degree between −180 and 180. The rotation point $z^* = (z_1^*, \ldots, z_n^*)$ for the point z is defined in the dimension i as follow:

$$u_i = z_i - (a_j + b_j)/2, \quad v_i = \sqrt{(z_i - a_i)(b_i - z_i)}, \tag{46}$$

$$u_i^* = u_i \times \cos\beta - v_i \times \sin\beta, \tag{47}$$

$$z_i^* = (a_i + b_i)/2 + u_i^*, \quad \beta = \beta_0.\, N(0, 1) \tag{48}$$

Then, RBL is embedded into DE with the same scheme as ODE. Park et al. [192] proposed a stochastic OBL using a beta distribution with the selection switching and the partial dimensional changing schemes which integrated it with a modified DE. By using the beta distribution, two types of opposition, convex and concave, were defined based on the parameters of the beta distribution. The concave opposite of the point x_i is defined with a beta distribution whose parameters α and β are both greater than 1 as follow:

$$\breve{x}_{i,j} = (b_j - a_j).\, Beta(\alpha, \beta) + a_j \tag{49}$$

$$\alpha = \begin{cases} spread.\, peak, & \text{if} \quad mode < 0.5 \\ spread, & \text{otherwise} \end{cases} \tag{50}$$

$$\beta = \begin{cases} spread, & \text{if} \quad mode < 0.5 \\ spread.\, peak, & \text{otherwise} \end{cases} \tag{51}$$

$$spread = \left(\frac{1}{\sqrt{normDiv}}\right)^{1+N(0,0.5)}, \tag{52}$$

$$peak = \begin{cases} \dfrac{(spread - 2)mode + 1}{spread(1 - mode)}, & \text{if } mode < 0.5 \\ \dfrac{2 - spread}{spread} + \dfrac{spread - 1}{spread.\, mode}, & \text{otherwise} \end{cases} \tag{53}$$

$$mode = \frac{(a_j + b_j - x_{i,j}) - a_j}{b_j - a_j} \tag{54}$$

where Beta(α, β) is a beta distribution with the parameters α and β, $N(0, 0.5)$ is a Gaussian distribution. The *normDiv* is the normalized diversity which is defined as follow:

$$normDiv = \frac{1}{NP}\sum_{i=1}^{NP} CD(x_i(t), POP(t)) \tag{55}$$

$$CD(x_i(t), POP(t)) = \min_{c \in POP(t), x_i(t) \neq c} d(c, x_i(t)), \tag{56}$$

$$d(c, x_i(t)) = \sqrt{\frac{1}{D}\sum_{j=1}^{D}\left(\frac{x_{i,j}(t) - c_j}{b_j - a_j}\right)^2} \tag{57}$$

The convex opposition is defined with different strategy of defining mode and spread such that α and β in the beta distribution should be smaller than 1. The mode and spread in the convex opposition is defined as follow:

$$mode = \frac{x_{i,j} - a_j}{b_j - a_j}, \quad spread = 0.1\sqrt{normDiv} + 0.9, \tag{58}$$

Also, the opposition is calculated as the partial dimensional scheme for some selected dimensions by utilizing the random strategy to choose the subset of dimensions. In addition, a selection switching scheme is employed such that if the population diversity is greater than a certain threshold, the best individuals are selected from the population and the opposite population via a method similar to $(\mu + \lambda)$ selection in the evolutionary strategy (ES). Also, if the diversity is smaller than the certain threshold, the worst half of individuals is replaced by their opposites, similar to (μ, λ) selection in ES.

3.1.6. Center-based sampling

As it can be seen from the definitions and theorems, the center point is a critical point Fig. 11. In [216], the intuitive explanation of the benefit of different opposite points was provided by using the center point in an interval. The concept of center based sampling was introduced by Rahnamayan and Wang [214,213,215]. They indicated that the probability of closeness for the center point to an unknown optimal solution is higher than other points and in the higher dimensions, this probability value increases extremely. A center-based region was introduced which includes points in the interval $[0.2, 0.8]$ in an interval $[0, 1]$. By using Monte-Carlo simulations, the probability of closeness to an unknown solution is calculated for different candidate solutions of the interval. Fig. 12 indicates the obtained results of Monte-Carlo simulations. In addition, they compared the center-point sampling with opposite points and confirmed that the center point has the higher closeness probability to an unknown optimum solution than the opposite points. Also, quasi opposite points are presented as a

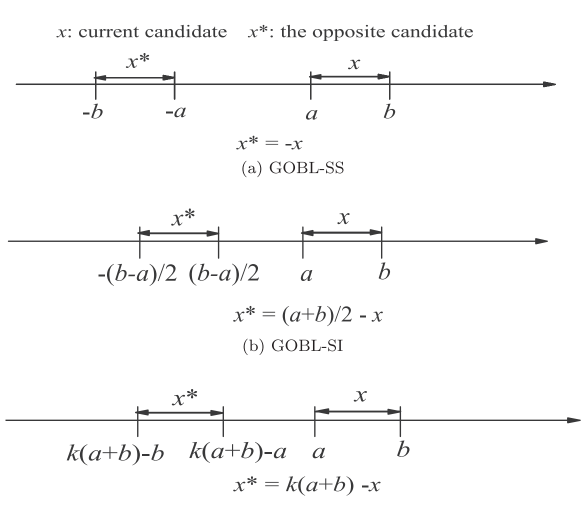


Fig. 11. Three schemes of GOBL.

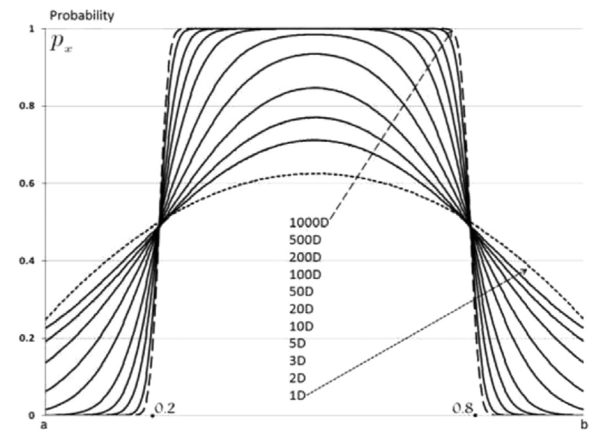


Fig. 12. Probability of closeness of candidate-solution to an unknown solution for different points in the interval [214]. Utilizing a vector with the same value for all its dimensions on the space diagonal is a proper way to provide 2D plots for the D dimensional search space.

promising and reliable evidence for supporting the proposed center-based sampling concept.

In [206], a new type of OBL was proposed according to the gravity center of population instead of using the minimum and maximum boundary of all candidate solutions in a population. In the centroid opposition-based different evolution (CODE), the centroid-opposite point for the candidate solution x_i is defined as follow:

$$\breve{x}_i = 2{*}M - x_i, \quad M = (x_1 + x_2 + \ldots + x_{NP})/NP, \tag{59}$$

where NP is the number of candidate solutions. In [355], Opposite-Center Learning (OCL) was proposed based on minimizing the distance of the pair including the original candidate and the opposite point to the global optimum. The opposite center point p_{oc} for the starting point $p_o = (x_1, x_2, \ldots, x_D)$ is defined as follow:

$$p_{oc} = \underset{p}{argmin} \int_{p_s \in \mathrm{T}} \| p - p_s \| f(p_s) dp_s, \ \mathrm{T} = \{p_s : \| p_o - p_s \| > \| p - p_s \|\}, \tag{60}$$

where p_s is the global optimal solution and $\|.\|$ is a distance metric which in [355] two distance metrics, Euclidean and squared, were considered.

3.2. Theoretical research works on RL

In [263], oppositional target domain estimation (OTE) was proposed to reduce the search and navigation area in the target domain estimation. In target domain estimation, the environment is an n dimensional grid and each cell of grid represents a state. An action changes the state coordinates by the value Δ. In OTE, two types of evaluative feedback are computed; reward and punishment. After taking action, if the agent is located at a smaller distance to the target, the agent receives the reward r; otherwise the agent receives the punishment p. The different types of opposition for the target domain estimation are defined as following.

Definition 8 (*Opposite State***).** [263] Suppose that the environment is an n dimensional state space ($\subset Z$) therefore each state can be represented as an D dimensional point $s(s_1, \ldots, s_D)$ and $b_i \leq s_i \leq c_i \quad \forall\, i \in 1, \ldots, D$. The opposite state is defined as follow:

$$\breve{s}_i = b_i + c_i - s_i \tag{61}$$

Definition 9 (*Opposite Action***).** [263] Suppose that an action changes the coordinates of a given state in a certain direction by the value Δ ($s' = s \pm \Delta$). The opposite action $\breve{s}$ is able to change the coordinates to the opposite direction by the same value ($s' = s \mp \Delta$).

An OTE theorem is proved which indicates when an agent in the state s receives the same evaluative feedbacks by taking an opposite actions in opposite states while at least for one action, the agent receives a reward, the target is located in a reduced sub-space between s and $\breve{s}$.

3.3. Theoretical research works on ANN

Ventresca et al. [311] developed mathematical proofs to investigate the effectiveness of employing opposite transfer functions. The theoretical analysis were extended to consider the benefit of opposite networks in two situations; network initialization and early stages of the learning process. They proved that OTFs are symmetrical transformations in the weight space and with supposing a minimal random network as the base case for the transformation, they represented a unique input-output mapping.

4. Developmental research works

This section presents those research works which focus on the design of strategies to employ OBL in the machine learning algorithms. Developmental research works are categorized in four classes: optimization, ANN, RL, and fuzzy system methods.

4.1. Optimization methods

Many optimization methods have been developed by utilizing the OBL concept in a) the initialization (population-level), b) during evolution, or c) designing mutation and crossover steps (operation-level). In this section, optimization algorithms are surveyed in three different classes evolutionary computation, swarm intelligence, and multi-objective optimization.

4.1.1. Evolutionary computation

In [309], a modified SA was proposed based on OBL scheme which computes both a neighborhood and an opposite neighborhood of the current candidate solution and selects the better one of them as the current candidate solution. In [205], two types of time varying jumping rate, linearly increasing and decreasing functions, were introduced. The varying jumping rates are designed in a special way such that during exploration, the linearly increasing jumping rate has the lower value and during exploitation it has the higher value and vice versa for the linearly decreasing jumping rate. The results indicated that the linearly decreasing jumping rate performs better that other jumping rates (including a constant value for the jumping rate). In [212,211], ODE was applied to solve large scale benchmark functions (i.e., D=500 and D=1000). The results confirmed that ODE can achieve better results than parent algorithm, DE. Ventresca et al. [310] introduced a new probability update rule and sample generation procedure based on OBL for the population-based incremental learning algorithm. OBL is applied to control and improve diversity and also, they gave some mathematical proofs, definitions and theorems to analyze the effect of diversity based on OBL on the performance of algorithm which are briefly described as follows. It is assumed that the distance between two samples be the Hamming distance therefore the diversity of the population ρ is defined as follow:

$$V(\rho) = \sum_{i=1}^{n}\sum_{j=1}^{i-1} d_{HAM}(p_i, p_j), \tag{62}$$

where p_i and p_j are samples of the population ρ. A comparison set S is defined as the cartesian product $A \times B = \{\langle a, b\rangle | a \in A \quad \text{and} \quad b \in B\}$. Let U denotes universal set and subset $A \in U$, they defined the opposite set as $\breve{A} = \{\langle a_i, \breve{a}_i\rangle | a_i \in A \quad \text{and} \quad \breve{a}_i \in U\}$; a_i and $\breve{a}_i$ are given candidate solutions and its opposite solutions $\in A$. The diversity of the opposite set $\breve{A}$ is defined as $\sum_{i=1}^{m} d_{HAM}(p_i, \breve{p}_i)$. In following, the concept of proved theorems are briefly described. It is mathematically proven that in the k samples including $k/2$ random candidate solutions and $k/2$ their opposite candidate solutions, diversity of opposite solutions is greater than only random candidate solutions. They assume that M is a probability matrix in d-dimensional binary space, $\Re^d$, and sets R_1, R_2, and $\breve{R}_1$ are in $\Re^d$ with $|R_1| = |R_2| = |\breve{R}_1|$, if $G_1 = R_1 \bigcup R_2$ and $G_2 = R_1 \bigcup \breve{R}_1$, then $V(G_2) \geqslant V(G_1)$. Finally, they applied the generated sampling based on an increased diversity strategy to the population-based incremental learning algorithm. A scalability test was conducted for GOBL used in DE algorithm in [330] on the benchmark functions with 50, 100, 200, and 500 dimensions. In [331], the diversity of ODE was analyzed to indicate how the generation jumping step of ODE increases the diversity to find more potential regions and the elite selection step of ODE accelerates the convergence speed. Two steps of ODE are divided into three states, before opposition, after opposition, and after selection. Then, the diversity of each state is computed to demonstrate their diversity. OBL was embedded in HS algorithm [90] to enhance the mutation operator of the HS. After generating a new candidate solution by mutation operator, the opposition of new solution is calculated and the best one of them is selected as a new candidate solution. In [184], the quasi-opposite of the worst harmony is computed to enhance a HS with a quadratic interpolation. In [352], they conducted a different strategy of using OBL such that for each candidate solution, after computing its opposite, the opposite solution is compared with only its parent solution to choose the better one of them. In [366], the opposite points were used based on combining multiple points [286] in the population initialization phase and during the evolution process with a jumping rate. The opposite point OP_i for an individual P_i is defined as $OP_i = 2x_{best} - (m_1 * P_i + m_2 * P_{i1})$, where x_{best} is the best candidate solution of the current population and m_1 and m_2 are two uniform random numbers such that $m_1 + m_2 = 1$. A centroid-based initialized JADE was proposed in [127] by using a centroid based strategy in the initial population. In the random initial population, three points are selected and their center is calculated as:

$$x_i = (x_1 + x_2 + x_3)/3, \tag{63}$$

and this process is continued for other three points and N_p center points are obtained. N_p center points are sorted corresponding to their objective values and 30% worst center points are selected as a subpopulation (SP). SP is divided to two subpopulations; SP1 including $\frac{1}{3}$ first worst points of SP and SP2 including $\frac{2}{3}$ second worst points of SP. Then, the individuals of SP2 are reflected through individuals of SP1 by the following formula:

$$x_{r,j} = x_i + 0.6. (x_i - x_1), \tag{64}$$

which x_1 is the first member of SP2 and x_i is the mean of first two individuals of SP1. Then, these reflected candidate solutions are replaced with the worst 10% of center points and the obtained population is considered as an initial population. In [79], a Center-Based Differential Evolution (CDE) was proposed based on a center-based sampling strategy. In CDE, the opposition of each individual in the population is calculated and the opposite population is constructed. Then, the center-based population is created by generating a random candidate solution between each candidate solution and its opposite. The NP fitness solutions from current population and the center-based population are selected as current population. This strategy was applied in the initial step and during evolution step with a predefined jumping rate. In [107], GOBL was applied to enhance the performance of the compact DE. When the trial candidate solution is generated, the opposition of offspring is computed by GOBL. The trial candidate solution and its opposite are compared and the best one would be selected as a candidate solution of the next generation. In [5], four schemes were proposed for using OBL in the shuffled differential evolution (SDE). In the first scheme, OBL is utilized with the same strategy like OBL scheme in ODE algorithm. The second scheme uses OBL as an extra step to improve the individuals in each memeplex in the SDE algorithm. In third scheme, in addition to using OBL in the process of the evolution, it applies OBL after a complete iteration of the evolution process in the SDE. The last version compares each individual with its opposite and selects the best one and also applies OBL after a complete iteration of the evolution process. A center-point-based SA was proposed in [80] in which center-point is utilized as the initial starting point for the search. In [385], the elite OBL (EOBL) was introduced which computes GOBL of the elite candidate solutions to compute the opposite point. In [375], OBL was employed to find the proper value for DE parameters, (F, CR), by using two pools to store values of parameters and their opposite values. Wang et al. [322] introduced a parallel DE algorithm (GOjDE) which employs GOBL to enhance its performance. GOjDE is implemented on the multiprocessors of GPU in parallel to decrease effectively the computational time. In [197], triple and quadruple comparison-based methods were designed to enhance the performance of DE algorithm by taking the benefit of using opposite points as well the paired comparison of the ordinary DE. Triple comparison methods include three comparisons: (1) the target, trial, and the opposition of trial candidate solutions, (2) the target, trial, and the opposition of target candidate solutions, (3) the target, trial, and random candidate solutions. In the quadruple comparison-based method, four candidate solutions are compared including the target, trial, the opposition of trial, and the opposition of target candidate solutions. Using opposite points with adaptive variants of differential evolution were considered in [305] and the results indicate that applying OBL into adaptive DE algorithms is not effective like non-adaptive DE algorithms. Pei et al. [196] applied OBL to tune an adaptive parameter of adaptive support vector regression (SVR) which approximates the fitness landscape to enhance the evolutionary computation. The tuned adaptive parameter by using OBL indicates the topological structure of the higher dimensional search space. In [91], the Harmony Search (HS) method with dual memory was proposed which uses OBL-based secondary harmony memory. The opposition concept for solving combinatorial problems such as graph-coloring and TSP was defined in [74,357]. Two type opposition concepts in the discrete space are defined based on the kind of a combinatorial problem: (1) for open path problems, in which their final node may be disconnected from the first node, such as the graph-coloring problem, or (2) for closed path problems, their final node is connected to the first node, such as the TSP. In the open path problems, first proximities among nodes are computed as the number of edges between two nodes. Based on proximities among nodes, the opposition of a path can be the path with the maximum proximity between adjacent nodes and the minimum proximity between further nodes. A greedy approximation of finding the opposite path was proposed which depends on the combinatorial problem. For closed paths, they proposed the opposite cycle path by representing a closed path on a circular path. After using a circular representation for the path, a clockwise direction (CW) of opposite path is defined as following.

Definition 10 (*The opposition of a closed path***).** [74,357] Suppose that P and n are an even node cycle and the number of nodes in a graph. The CW opposite path (P_0^{CW}) is defined as:

$$p = [1, 2, \ldots, n], \tag{65}$$

$$P_0^{CW} = [1, 1 + n/2, 2, 2 + n/2, \ldots, n/2 - 1, n - 1, n/2, n] \tag{66}$$

In [242], a DE algorithm based on Type-II OBL concept, $\breve{x}_{II}$, was proposed by using the center-based interpolation. In the initialization step, the interpolation employs the computed objective functions of the initial population to estimate the corresponding Type-II variables. During evolutionary process, it is updated by using the examined candidate solutions in the optimization algorithm. Also, they compared two opposition methods; min-max and centroid. By Monte-Carlo simulation, it was indicated that the centroid-based opposition can be used for Type-II OBL and also it performs better than min-max-based opposition according to the computed probability of their success. Type-II and Type-I opposition methods are utilized in the initialization and the evolutionary process of a DE algorithm like similar scheme used in ODE but the fittest solutions are selected from union of x, $\breve{x}$, and $\breve{x}_{II}$. In [120], the OBL concept was integrated into the cooperative co-evolutionary (CC) algorithm to solve large scale problems. In the CC framework, when a subcomponent of variables is optimized, the opposition of variables in the subcomponent is calculated for a candidate solution and other remaining variables are constant.

In [149], an enhanced GODE algorithm, AGODE, was introduced by using an adaptive GOBL approach. An adaptive parameter of jumping rate, p_0, is defined according to the success rate of the opposition operator. When the opposition obtains the better solutions, the p_0 value is increased; otherwise it is decreased. In [47], OBL, chaotic search, and quantum methods were combined in DE to determine a proper selection for the crossover factor, scaling factor, and mutation operator. Qin et al. [203] utilized OBL in a HS which performs independent HS with respect to multiple random groups. Deng et al. [58] proposed a local search to increase the performance DE which uses an orthogonal crossover [137,340]. Some trial candidate solutions are generated by the orthogonal crossover and then their quasi-opposites are computed and the best one from all generated trial solutions is selected. In [351], in each iteration, the population is randomly divided into three subpopulations and then DE, OBL, and GSA are applied on only one of three subpopulations to generate the new trial candidate solutions. In [169], the uniform randomly candidate solutions and their opposite were used in a surrogate-assisted DE to enhance the off-line training for constructing surrogate model. In [327], some experiments were conducted to compare the performance of different ODE schemes. The dimensions 10, 30, and 50 were considered and results indicated that based on the number of winners, QODE outperforms other algorithms for D=30 and ODE outperforms other algorithms for D=50. Kalra et al. [114] proposed using ANN to compute Type-II OBL. An ANN is employed to discover relation between the candidate solution x and its type-II opposite $\breve{x}_{II}$ as input and output of the network. The network is trained on training data, then it can be used to predict type-II opposite. Mahdavi et al. [155] introduced a CC framework with the population initialization strategies based on the center-based sampling. Center-based normal distribution sampling, central golden region, and hybrid random-center normal distribution sampling strategies are utilized for the population initialization. In center-based normal distribution sampling strategy, a normal distribution with the mean value of the center point is used to generate initial candidate solutions. Central golden region sampling generates initialize candidate solutions close to the center point by limiting the search space to the middle 60% interval of the search space. In hybrid random-center normal distribution sampling, the 50% of initial population is generated with the normal distribution sampling strategy and another half is randomly sampled by using the uniform distribution. In [338], a GA algorithm with the variable neighborhood search was proposed to solve the two-stage assembly flowshop scheduling problem. For four neighborhood structures; insert, swap, exchange, and inverse; their opposites are generated by changing each gene value (g_i) for each candidate solution to $1 + n - g_i$ which n is the number of jobs in a scheduling problem.

4.1.2. Swarm intelligence

In [38], an enhanced PSO was proposed which uses OBL for some worst personal particles to replace with their opposites. OBL is applied to create a personal best opposite in PSO and the best one from personal best and its opposite is selected as personal best [95]. In [118], the super-opposition concept was integrated into a PSO with an adaptive velocity to enhance its exploration ability. A re-initialization method according to the super-opposition concept is activated to avoid premature convergence when stagnation and premature convergence states are identified. Each particle $P_s(t)$ in a swarm with a super-opposite point $\breve{P}_s^{(sup)}$ is generated randomly from a uniform distribution on the following range:

$$\breve{P}_s^{(sup)} \in \begin{cases} [l_d(t), \breve{P}_{sd}) & P_{sd}(t) > (u_d(t) + l_d(t))/2 \\ [l_d(t), u_d(t)] \setminus \{P_{sd}(t)\} & P_{sd}(t) = M_d(t) \\ (\breve{P}_{sd}, u_d(t)] & P_{sd}(t) < M_d(t), \quad d = 1, \ldots, N \end{cases} \tag{67}$$

Where $\breve{P}_{sd}$, $l_d(t)$, and $u_d(t)$ are the opposition of $P_s(t)$, the minimum and maximum values of the *d-th* dimension in the swarm particles at iteration t, respectively. In [180,182], the candidate solution with the lowest fitness was replaced by its opposite in PSO algorithm. OBL is combined with lévy flight ABC in [249] which opposition of swarm is computed after applying scout bee phase in the solution search process based on a jumping rate. In [151], OBL was embedded to an orthogonal learning PSO which computes the opposite position of the worst particle in the swarm. The OBL concept was integrated into the updated equation of particle in the PSO algorithm in [147]. The PSO equations are modified as follow:

$$v_{ij}(t+1) = w.\ v_{ij}(t) + r_1.\ (Pbest_{ij}(t) - x_{ij}(t)) + \tag{68}$$

$$r_2.\ (Gbest_{ij}(t) - x_{ij}(t)) + r_3.\ (Rworst_{ij}(t) - x_{ij}(t)), \tag{69}$$

$$x_{ij}(t+1) = x_{ij}(t) + v_{ij}(t+1), \tag{70}$$

$$r_1 + r_2 + r_3 = 1, \quad r_1,\ r_2,\ and\ r_3 \text{ are random number} \in [0, 1], \tag{71}$$

where $Rworst_i$ is the worst $Pbest$ solution of all $Pbest(t)$ solutions at iteration t. In [45], OBL was applied in PSO algorithm such that each dimension of a candidate solution is disturbed by its opposite value with a specific disturbance rate. Zhou et al. [382] proposed an opposition-based learning competitive particle swarm optimizer which uses OBL to avoid premature convergence in PSO. Three particles of swarm are randomly selected and compared with each others which are called the winner, neutral, and loser candidate solutions based on the descent arrangement of their fitness. The winner particle is passed without any change to the next iteration while the position and velocity of the loser particle are updated by learning from winner, then it is passed to the next iteration. The position and velocity of the loser particle are modified by computing their opposite, then it is passed to the next iteration. An initial population was generated for the ABC algorithm by using OBL and chaotic systems in [86,84,85,65,363]. First, chaotic candidate solutions are generated by employing chaotic systems and then their opposite are computed and the fittest individuals from the union of chaotic and opposite solutions are selected as the initial population. In [386], the GOBL concept was employed in the Scout Bee phase to generate new food sources. An ABC algorithm by using center-based sampling strategy was proposed in [39]. The centroid of population is defined as:

$$C = \frac{\sum_{i=1}^{SN} X_i}{SN}, \tag{72}$$

where SN is the number of the food sources and X_i is the position of the *ith* employed bee. Then, the centroid food source is applied in the exploration step. In [370], OBL was applied to the worst firefly to find better firefly.

A scouting predator-prey optimizer was proposed in [266] by using various types of scout particles. A type of scout computes the opposition of the worst particle to discover a promising search region. A modified ABC by using opposition-based learning was proposed in [133] which it generates new candidate solutions for the employed and onlooker bees by using OBL to select the fitter candidate solutions. Using the OBL concept for ACO was investigated in [164,162]. They introduced five different schemes to apply OBL to improve the construction phase of ACO by utilizing synchronous opposition, free opposition, free quasi-opposition, opposite pheromone per node, and opposite pheromone per edge. In [163], two extension steps were proposed to use the opposition for ACO; the opposite pheromone per node (OPN) and the opposite pheromone update (OPU). OPN was proposed to escape from a local optimum which it changes the pheromone of ants to move them to the opposite direction of their current paths. In this method, the next city is picked up according to the original pheromone content τ or the opposite pheromone content $\breve{\tau}$ with a random value. For an edge between the current node i to an available node j, $\breve{\tau}$ is computed as $\breve{\tau} = \tau_{min} + \tau_{max} - \tau$. OPU computes the opposite of the pheromone update phase in the ant algorithm. It provides additional updates to modify the pheromone content faster than the basic ACO. The opposite update is calculated based on a random value. The opposite rating and the opposite pheromone content for all edges are defined as follow:

$$\breve{o}_{ij} = \frac{|\mu_{ij} - \mu_i^{bs}|}{\mu_{max} - \mu_{min}}, \tag{73}$$

where μ_{ij} indicates the heuristic function value for the edge going from the node i to the node j, and μ_i^{bs} is the value for the edge outgoing from the node i included present in the best path. The values μ_{max} and μ_{min} denote the maximum and minimum heuristic values of the graph.

$$\tau_{ij}^{new} = \tau_{ij}^{current} - \sum_{k=1}^{m} \breve{o}_{ij} \Delta\tau^k, \tag{74}$$

where $\tau_{ij}^{current}$ is the current pheromone level on the edge going from the node i to the node j, $\Delta\tau^k$ is the additional pheromone by the ant k. In [225,226], the full opposite combinatorial concept was introduced in which the search process of an ACO algorithms is divided into two steps; learning and solving by original ACO algorithm. In first step, an ACO algorithm is applied to maximize the combinatorial objective function while in the second step, the ACO algorithm deals with the combinatorial objective function to be minimized. The main motivation of the learning step is extracting knowledge in the opposite direction by keeping the worse solutions to find some search region which is transmitted into the second step for using in the ACO algorithm. Also, in the first step, the heuristic knowledge of the worst candidate solution is modified by computing its opposite which gives it more priority during the optimization process. Also, the OBL concept has been employed in the initialization step of several optimization methods [87,113,113,332,167,130,349,191,316,314,379,189,3292,250,290,350,146,181,317,111,110,185,81]. In addition, some optimization algorithms used the OBL concept only in updating steps of candidate solutions [315,150,247,319,333,179,40,183,168,152,99,99,-334,384,138,37]. Several algorithms have utilized the various concepts of OBL by the proposed scheme in [210] in their initialization and during updating steps [246,108,109,339,285,115,318,166,347,374,324,65,70,71,180,378,321,153,273,133,34,145,343,82,160,353,325,346,336,103,12,4341,369,200,304,335,16,317,112,53,219,354,323,371,52,208,100,373,174,172].

4.1.2.1. Multi-objective optimization. In the multi-objective optimization methods, also the OBL concept was utilized to enhance their performance. Some research works applied OBL in their population initialization phase [198,291,193,10]. In [48,50], OBL is incorporated into self-adaptive mechanisms for the probability of the mutation and crossover in DE and also is hybridized with multi-objective evolutionary gradient search to develop a local search. In [51], a novel grid-based DE variant for many objective optimization was proposed which utilizes the opposite of the mutation factor F to increase the probability of finding a proper value for the mutation factor. OBL was used in the population initialization and during the evolution process of multi-objective DE to enhance its performance in [63]. OBL was applied to the personal best positions in order to guide the search process in the multi-objective PSO algorithm [94]. In [154], the center-based opposite points and GOBL were applied in the initial population and during the evolution process of DE, respectively, to enhance the performance of MOEA/D algorithm. Leung et al. [136] proposed a new strategy based on OBL to find the proper parameter values for the multi-objective DE algorithm (MODEA). In this method, the opposition of those parameters are computed which they do not have proper values to generate better candidate solutions.

A uniform weight vector generation method was proposed in [194] for decomposition based multi-objective genetic algorithm (DMOGA) which uses OBL to generate weight vectors. First, the range [0, 1] is linearly divided into N values as weights for the first objective function; then their opposite of these values are also calculated as weights for the second objective function. Two opposition-based competitive coevolution multi-objective optimization algorithms [284] were proposed based on two competitive fitness strategies, namely, the Hall of Fame (HOF) and K-Random Opponents (KR). In these methods, the competitive fitness strategies select individuals as the opponents which simulate the OBL concept. An opposition-based competitive coevolution for a set of individuals $(P(I_1, I_2, \ldots, I_m),\ I_1, I_2, \ldots, I_m \in I)$ in the population with size NP is described by defining the opposite set as $\breve{P}(\breve{I}_1, \breve{I}_2, \ldots, \breve{I}_m)$ with $\breve{I}_i = I_i - I$ which the operator "−" denotes the operation "remove from" operation. Then, the opposition-based competitive coevolution selects the fittest from the individuals of two sets. In the KR strategy [190], each individual competes with K other individuals of opponents. In the HOF strategy [227], each individual is competing against every archived best individual from previous generations. The proposed opposition-based coevolutionary algorithms uses SPEA2 as multi-objective optimization algorithm which is integrated with HOF and KR strategies. In SPEA2-CE-HOF and SPEA2-CE-KR methods, individuals are selected as the opponents by utilizing HOF or KR strategies, respectively. In [283], OBL is used to improve both convergence rate and distribution of Pareto Front (PF) solutions in evolutionary multi-objective optimization algorithms. During the evolutionary process, the opposite points are generated which are closer to true PF and also well-distributed; the opposite points try to fill out the sparse region (i.e., higher crowding distance area) of the PF by translating of the points to those regions. In fact, the opposition-based jumping helps to have new individuals closer to first front and also well-distributed. Assumption is that, the translating of bad individuals in the decision variable space (Type-I opposition) is corresponding to move toward the rank first PF curve in the objective space based on an implicit Type-II opposition scheme.

4.2. Artificial neural networks (ANN)

In [308,312], Ventresca and Tizhoosh used the concept of opposite transfer functions for back-propagation ANN through time algorithm (BPTT) to enhance learning in the BPTT algorithm. Transfer functions of network, the original transfer function or its opposite, are utilized is based on a probabilistic estimation procedure. A probability for each hidden neuron is computed according to the error function of the network. If network obtains a lower error, the probability of the same transfer functions in the network will be increased; otherwise, it will be reduced. In [313], the concept of OTFs was utilized in the large-scale neural networks including thousands of parameters. In learning step, the transfer function of some neurons were changed to their opposite transfer function according to a probabilistic rule in each iteration. The network including the opposite transfer functions represents the opposite network. Then, both current and opposite networks were examined and the best one of them is selected as the current network. The OBL concept was modified and applied to DE algorithm for designing of a beta basis function neural network (BBF) [59]. The opposition of the point $X(x_1, x_2, \ldots, x_D)$ is defined by:

$$\breve{x}_i = \begin{cases} \alpha_i((a_i + b_i)/2 + x_i) & x_i < (a_i + b_i)/2 \\ \alpha_i((a_i + b_i)/2 - x_i) & x_i > (a_i + b_i)/2 \end{cases} \tag{75}$$

where α_i is a random number in [0, 1]. Then, it is applied during initialization step. Also, in mutation and crossover operators, it is also utilized as follow:

$$\breve{v}(i) = \breve{x}_{r1}(i) + F(x_{best}(i) - \breve{x}_{r2}(i)) + F(\breve{x}_{r3}(i) - \breve{x}_{r4}(i)), \tag{76}$$

$$\breve{u}(i) = \begin{cases} \breve{v}(i) & rand(0, 1) < Cr \\ \breve{x}(i) & \text{otherwise} \end{cases} \tag{77}$$

Where CR and F are the predefined crossover and mutation rates and $\breve{x}_{r1}(i), \breve{x}_{r2}(i), \breve{x}_{r3}(i), \breve{u}(i)$, and $\breve{v}(i)$ are the opposition of $x_{r1}(i), x_{r2}(i), x_{r3}(i), u(i)$, and $v(i)$; respectively. A hybrid improved opposition based PSO was proposed in [361] to optimize the weights of ANN prediction models. To improve the search ability, the opposite position and velocity of each particle are calculated in the initial population and during iteration of PSO and the best one is selected from the particle and its opposite. For training a feedforward neural network (FNN), a modified PSO was applied in [222], which uses opposition-based initialization and opposition-based generation jumping concepts of OBL. Also, in [60,61] the defined OBL concept in [59] was applied to a PSO algorithm for the design of a beta basis function neural network (BBF). OBL is applied in the initialization step for particles and then the fittest particles are selected from particles and their opposites. In the update position of particles, the opposition of particle $\breve{p}$ for each particle p_i with velocity v_i is also defined as follow:

$$\breve{p}_i = \breve{p}_i + v_i, \tag{78}$$

In [360], a hybrid PSO algorithm with applying OBL in the initialization step was proposed to train feedforward neural network. In [188], GSA was combined with OBL for training feedforward networks with the weight decay. In [312], opposite transfer functions were proposed to improve the numerical conditioning of neural networks and also to extrapolate two back-propagation-based learning algorithms.

4.3. Reinforcement learning (RL)

In [259], opposition-based $Q(\lambda)$ algorithm ($OQ\lambda$) was proposed based on the eligibility traces. They introduced the opposite traces as the eligibility traces for opposite actions. In $OQ\lambda$, it is assumed that the agent receives the punishment $\breve{r}$, when takes opposite action $\breve{a}$ in the state s, therefore based on $\breve{r}$, the Q-matrix is updated for all states s and opposite actions $\breve{a}$ as follow:

$$Q(s, \breve{a}) \longleftarrow Q(s, \breve{a}) + \alpha\delta_2 e(s, \breve{a}), \tag{79}$$

$$\delta_2 \longleftarrow \breve{r} + \gamma Q(s'', a^{**}) - Q(s, \breve{a}), \tag{80}$$

$$a^{**} \longleftarrow argmax_b Q(s'', b), \tag{81}$$

where s'' is the next state after taking the action $\breve{a}$ and $e(s, \breve{a})$ is opposite traces for states s and opposite actions $\breve{a}$. Also, λ is a discount factor. In [260,261,258], the $OQ\lambda$ algorithm was modified by using non-markovian update of the opposite traces; called $NOQ\lambda$. An opposite weight W is defined to update opposite traces and the updating formula in the Eq. (80) is modified as follow:

$$Q(s, \breve{a}) \longleftarrow Q(s, \breve{a}) + W \times \breve{r} \times e(s, \breve{a}) \tag{82}$$

The new update formula does not depend on the next state. Tradeoff between exploration and exploitation in the $NOQ\lambda$ algorithm was investigated in [262,261,258]. An increasing weight function was proposed to improve the $NOQ\lambda$ algorithm. By increasing learning progresses, the opposite value of the weight (W) is gradually increased in order to increase the positive effects of the opposite for the update of Q-values ($Q(s, \breve{a})$). In [345], an active exploratory Q-Learning was proposed which at a state, a number of actions are considered and then the action with the greatest immediate reward is selected. The state of the selected action is considered as the next state. The opposition-based random search for the good action in the (OBR-SGA) method is proposed which generates the pool of actions based on OBL. If the random action can not obtain a good reward so the opposite of this action is examined. Also, they proposed opposition-based cyclic parameter adjustment-SGA (OBCPA-SGA) based on cyclic parameter adjustment [128] and OBL. In OBCPA-SGA method, in the current dimension, an action is changed to a pair of opposite action elements (a and $\breve{a}$) until the better reward cannot be reached. In [157], type-II opposition was employed in Q-Learning to obtain the operating policies in reservoir management. A trained function approximation is computed and updated by using the obtained knowledge from the main agent. A multi layer perceptron is used as the function approximator. The opposite agent does not take an action but it extracts a set of new action-value functions.

4.4. Fuzzy systems

In [301], a new method for image thresholding was proposed by utilizing the opposite fuzzy set. First, a window is considered around the center of an object and its size is iteratively increased. In each iteration, the opposition of dark pixels as fuzzy set is computed for the current window. The location of windows with the maximum difference of entropy between the two fuzzy sets are calculated. Then, the image threshold value is calculated based on the average of representative numbers of both fuzzy sets of the selected window. In [300], a method for the approximation of type II opposite was proposed via evolving fuzzy inference systems. In this method, a Takagi-Sugeno, or T-S fuzzy inference system [282] is used in which the fuzzy rules are described as follow:

$$\begin{aligned} &\text{IF} \quad x_1 \text{ is } \mathrm{A}_1 \text{ AND } x_2 \text{ is } \mathrm{A}_2 \text{ AND } \dots \text{ AND } x_n \text{ is } \mathrm{A}_n \\ &\quad \text{THEN} \quad y = f_j(x_1, x_2, \dots, x_D), \quad j = 1, 2, \dots, D, \end{aligned} \tag{83}$$

where x_i, y, and A_i are the input and output variables, and extracted fuzzy rules, respectively. To calculate Type-II opposite, a training data is sampled in which the input variables are some solutions and their corresponding objective function and the output variables are Type-II opposites of solutions. After extracting fuzzy rules, the Type-II opposites for new inputs can be approximated. Also, fuzzy rules can be refined to better approximate Type-II opposition by using future data points.

5. Contributions of OBL in application domains

In recent years, there has been a growing number of research works which apply the OBL concept in the machine learning algorithms to solve a variety of problems, such as power systems, pattern recognition and image processing, identification, bioinformatics, medicine, etc. OBL was applied in the optimization in power systems which have several objectives with equality and inequality constraints such as active power loss, fuel cost, etc. Also, it was used in optimization methods for solving image processing and pattern recognition tasks which require parameters tuning. In addition, OBL was utilized into optimization methods to find the best parameter values for the parameter identification of challenging nonlinear systems. The main application of OBL is in the bioinformatics and medicine field including disease diagnosis and prediction in molecules and proteins. Most utilized schemes of OBL in various application domains are the basic OBL concept and Quasi opposite. Table 2 summarizes the major applications of OBL; about 168 papers.

6. Conclusion remarks and future directions

OBL can potentially be used with the soft computing techniques to solve engineering and science problems. In this paper, a general overview of research works on OBL has been surveyed. Beginning with a comprehensive background of the basic OBL concept, it explained the different schemes and mathematical theorems of OBL in the machine learning algorithms. Next, it provided an extensive review of the modifications of using OBL in the reinforcement learning, artificial neural networks, fuzzy systems, and variant optimization methods. It gave a brief overview of most various applications of OBL. The content of the paper indicates that taking advantage of the OBL concept has various main aspects; (1) defining or selecting an appropriate scheme of OBL corresponding to an especial problem, (2) recognizing how OBL can be used and which part of algorithm, (3) verifying the efficiency of using OBL by mathematical theorems or empirically validating by developing comprehensive experiments. We define future trends of OBL and describe some following research gaps which have been identified.

- Solving high dimensional problems: Most of the current research works on OBL have focused on enhancing the performance of optimization methods in low dimensions. Much more effort is required to employ the advantage of OBL to further improve the scalability of the optimization methods to solve efficiently high dimensional problems.
- The untouched perspectives in the optimization methods: Although a number of research works have been proposed to use OBL in the optimization methods, the different perspectives of optimization methods are still greatly needed to be considered the advantage of employing OBL. The most of major directions are different type of variables such as integer, discrete, and mixed-type; other kinds of optimization problems, such as noisy, dynamic, combinatorial, multi-level, constraint optimization methods; strategies of the increase diversity, local searches, developing escaping methods from stagnation and local optimum; tuning methods of the control parameters; and landscape analysis.
- Self-adaptive and adaptive opposition-based methods: The most of OBL methods employ OBL with a jumping rate. A great potential to future research work can be developing adaptive and self-adaptive opposition-based algorithms which are able to control utilizing of opposition during searching, learning, or optimization corresponding to the type of problem.
- The uncovered fields in the machine learning: Using OBL is an effective method to enhance variant branches in the machine learning. It would be interesting to apply OBL to other fields of the machine learning methods such as big data analytics, clustering, classification, deep learning, sampling method, or metamodeling methods.
- A comprehensive comparison of variant opposition-based schemes and their corresponding algorithms: An extensive comparative study becomes an essential requirement to understand real capabilities provided by variant opposition-based schemes and corresponding algorithms and benefits, weaknesses, and limitations.
- Utilizing OBL schemes to accelerate mathematical or classical optimization methods: Most of meta-heuristic algorithms have been proposed based on using OBL. Further research can be directed to apply OBL to accelerate the performance mathematical or classical optimization methods (derivative or direct search based techniques).
- Utilizing OBL schemes in the positive and negative ways: Future research works can lead to investigate both positive and negative ways of using OBL schemes. With using OBL, algorithm can be able to identify which way enhances the performance of algorithm (for instance increasing diversity or decreasing diversity and selected (good ones) or removing some candidate solution (bad ones)).
- Utilizing OBL in the interactive optimization methods to reduce the number of user assessments: In the interactive optimization methods, user interacts significantly during the candidate solutions evaluation process. A great to future research works can be investigating of using OBL for the interactive optimization methods, for example using the intermediate optimization results and their opposites to get better feedback from user.
- The design of new opposition-based schemes of mutation, crossover, and selection operations: The most developed opposition-based

Table 2
Summary of applications of OBL.

Sub Areas and Details	Algorithms and references
Power Systems	
Reactive power dispatch	Quasi-oppositional DE [21], quasi-oppositional teaching learning based optimization [165], opposition-based GSO [253], opposition-based self-adaptive modified GSO [178], quasi-oppositional BBO [231], opposition-based GSO algorithm [252], opposition-based improved PSO [33], improved opposition-based HS [135], quasi-oppositional harmony search algorithm [257]
Economic dispatch	Oppositional krill herd algorithm [23], multi-objective quasi-oppositional teaching learning [230], opposition-based GSO algorithm [251], opposition-based HS [35], opposition-based GSA [20], oppositional real coded chemical reaction optimization [26], DE with OBL, initial population [279], oppositional BBO [29,28], quasi-oppositional GSA [32], OBL improved DE [280], opposition-based GSA [187], opposition based DE [223], hybrid fuzzy-opposition based DE [294], opposition based DE [295], oppositional teaching learning based optimization approach [233]
Electrical power distribution system	Oppositional krill herd algorithm [278], opposition based DE [176], multi-objective quasi-oppositional teaching learning based optimization [277], quantum mechanics DE [46], hybrid Fuzzy-opposition based DE [175], ODE [131], ODE [177]
Optimal power flow	Oppositional krill herd algorithm [170], a modified flower pollination [73], oppositional BBO [233], non dominated sorting multi objective opposition based GSO [30], opposition-based DE [42], quasi-oppositional BBO [229]
Optimal design of power system stabilizer	Oppositional GSO [195]
Intelligent controller for load-tracking performance of an autonomous power system	Opposition based GSA [17]
Protection relays in a power system	Opposition based chaotic DE [11]
Isolated wind-diesel hybrid power system model	A novel opposition-based GSO [17]
Solution of unit commitment problem	Quasi-oppositional teaching learning based algorithm [234]
Short-term hydrothermal scheduling problems	Oppositional real coded chemical reaction based optimization [26]
Hydrothermal power system	Quasi-oppositional GSA [22]
Hybrid power system	Quasi-oppositional HS [289,288], quasi oppositional HS [158], quasi oppositional HS [159]
Automatic generating control	Quasi-oppositional HS [255], quasi-oppositional HS [254], quasi oppositional HS [256], oppositional BBO [228]
Load frequency control of multi-source multi-area power system	Quasi oppositional HS algorithm [248]
Coordination of directional overcurrent relays	Opposition based chaotic DE [36]
Pattern Recognition and Image Processing	
Clustering	Improved cat swarm optimization algorithm based on OBL [132]
Feature selection	Opposition chaotic fitness mutation based adaptive inertia weight binary PSO [25]
Face recognition	Opposition PSO with support vector machine [101]
Image Processing	Opposition-Based RL [239], opposition-based RL [238], opposition-based RL [125], Micro opposition-based DE [207], thresholding algorithm by utilizing the concept of opposite fuzzy sets [8], GA by using OBL and self organizing map based fuzzy hybrid intelligent method [126], OBL-based cooperative PSO [380], modified ABC [24]
Identification problem	
Parameters identification of solar cell models	Improved free search DE by using OBl for the worst solution [13], generalized oppositional teaching learning based optimization [275]
Nonlinear system identification	Opposition based DE for training neural networks [275], ODE combined with Levenberg Marquardt [276]
Parameter identification of hyperchaotic systems	Oppositional backtracking search optimization [143]
Identification of coupled pitch and heave motions	Opposition-based PSO [56]
Parameter identification problems on graphics hardware	PSO by using GOBL concept [337]
Traffic congestion identification	Opposition-based RL based on fuzzy C-means clustering [367]
Identification of fuzzy inference systems	Multiobjective opposition-based space search algorithm [106]
Parameter identification of uncertain fractional-order chaotic systems	A hybrid ABC by using chaotic opposition searching method [104]
Bioinformatics and Medicine	
Sorting unsigned genomes	Memetic and opposition-based learning GA [55]
Breast cancer diagnosis	Opposite weight back propagation NN classifiers [241]
Diagnosis of cardiac disease	The neuro fuzzy ECG classification network by using oppositional BBO [186]
Diagnosis in mammography images	Opposition-based classifier [240]
Molecular docking	Opposition-based DE [129]
Protein structure prediction	Improved PSO with OBL [381]
Medical image contrast enhancement	Opposition-based firefly algorithm [68]
Recognition of maize Leaf diseases	An improved PSO algorithm for neural networks [287]
Reconstruct organ boundaries in the human thorax using electrical impedance tomography	An oppositional BBO [221]
Classification of benign and malignant masses based on Zernike moments	Generic back propagation learning rule [281]
Optimization of feeding profile for an industrial scale baker's yeast fermentation process	Adaptive opposition based on differential evolution [372]
Aerobic fermentation process	Optimization methodology based on neural networks and self-adaptive DE [69]
Miscellaneous	
Network count location problem	Opposition based colonial competitive algorithm (OCCA) [14,15],
Infinite impulse response (IIR) system	Opposition based HS [306], opposition-based BAT algorithm (OBA) [237], opposition aided cat swarm optimization [62], hybrid DE [269], teaching-learning opposition based optimization [271], opposition based DE [265], hybrid DE by initializing OBL [268], opposition-based DE and binary successive approximation-based evolutionary search algorithm [264], enhanced teaching learning-based optimization [270]
Flow shop scheduling problem	An improved shuffled complex evolution algorithm with OBL [376], a new Cuckoo search algorithm with hybrid strategies [326], a hybrid discrete biogeography-based optimization (HDBBO) with using

(continued on next page)

Table 2 *(continued)*

Sub Areas and Details	Algorithms and references
	OBL [142], opposition-based DE [140], hybrid DE with OBL and bottleneck heuristic algorithm [204], opposition effective GSA based memetic [383], A new cuckoo search algorithm with hybrid strategies with GOBL [326]
Text summarization	ODE [2]
Operation cost minimization of a Micro-Grid	Using quasi-oppositional swine influenza model [25]
Non-metric lens distortion correction	The opposition learning-based PSO [41]
Electromagnetic parameters of the permanent magnet synchronous machine system	Improved comprehensive learning PSO with OBL [102]
Identification of critical to quality characteristics for complex products	Improved GSA by using OBL [320]
Forecasting agriculture water consumption	A novel BP-NN by using PSO with OBL for training [365]
Transistors of the differential amplifier circuit	Opposition based HS [161]
Knapsack problems	The binary version of firefly algorithm [27]
Service composition optimization	DE with OBL [224],
Optimized operation of microgrid	Improved GSA by using OBL [139]
Resource constrained project scheduling problems	DE with OBL [78]
Optimal classification of epileptic signals	A modified Hs based on OBL [88]
Optimal design of circular and concentric circular arrays with improved far-field radiation characteristics	Opposition-based BAT algorithm [220] (both initialize and jr)
Multi-UCAVs targets assignment	Opposition-based genetic algorithm [342]
Multi-objective optimization of retaining walls	A hybrid adaptive gravitational search algorithm [123]
Design and economic investigation of shell and tube heat exchangers	Intelligent tuned HS [303]
Optimal design of a PID controller	Opposition-based discrete action RL automata algorithm [201]
Time environment impact trade-off problem in construction projects	Opposition-based multiple-objective DE [43]
Optimal short-term hydro-thermal scheduling	Quasi-oppositional teaching learning based optimization [235]
Circle detection	An opposition-based chaotic GA/PSO hybrid algorithm [64]
Mobile robot controllers	Hardware opposition-based PSO [173]
Engineering optimization problems	An opposition-based HS [18]
Billboard advertising modeling	Opposition based colonial competitive algorithm [15]
Voltage rise mitigation	Enhanced opposition-based firefly algorithm [344]
Gun control system	Improved multiobjective DE [83]
Water resources problems	A hybrid ACO and DE using OBL [9]
Visualization of hidden structures in corporate failure prediction	A modified ACO algorithm by using obl [19]
FM matching synthesis	Opposition-based shuffled PSO with the passive congregation [171]
Design strategy of low-pass FIR filter	Opposition-based DE, [243]
Path planning for unmanned air vehicles	An improved ABC [134],
Epileptic EEG signal Classification	A modified HS based on OBL [89]
Rolling schedules for tandem hot rolling	Opposition learning multi-objective GA [141]
An optimal design of coordinated proportional-integral	Modified teaching learning based optimization with OBL [293]
Placement of radio frequency identification	Opposition-based learning estimation of distribution algorithm with gaussian copulas [92]
Wireless sensor network optimization	Velocity-free multi-objective PSO with centroid by initializing swarm with OBL [93]
Generation expansion planning problem	Opposition-based DE [117]
Wireless access networks	A new modified BBO algorithm with the partial opposition-based learning [98]
Wireless sensor network dynamic deployment	A modified ABC with OBL [6]
The design of digital differentiator	A hybrid optimization method with OBL [272]
Speed regulation in a chopper fed direct current motor drive	Opposition based artificial bee colony [218]
Design and economic investigation of shell and tube heat exchangers	ABC with OBL [303]
Tuning chess evaluation function parameters	ODE [31]
Truss topology optimization	Accelerated multi-objective particle swarm by using OBL in the initialize step [72]
Management of water resources	Opposition-based RL [156]
Symbolic regression	Balanced cartesian genetic programming by using Quasi OBL [368]
The neural modeling of a depollution process of some gaseous streams	Hybridization of the self-adaptive DE algorithm by initializing with OBL [54]
Earth slope stability evaluation in geotechnical engineering)	Opposition-based firefly algorithm [124]
The Location of median line in 3-D space	Teaching learning opposition based optimization method [217]
The spot color matching	A hybrid BBO and HS with OBL [144]
Running gait for humanoid robot	Opposition-based learning PSO [364]
Optimize ready-mixed concrete truck dispatch schedule	Integrating chaotic initialized opposition multiple-objective DE [44]
Investigation on the inversion of the atmospheric duct	ABC based on OBL [362]
TSP problems	Oppositional BBO [356,49]
Frequency modulation parameter optimization problems	Generalized opposition-based DE [237]
The nuclear reactor core design	DE with initializing OBL [236]
Short term hydrothermal scheduling problems	Quasi-oppositional GSA [232]
A decision-making process for multi-period portfolio problem	Quasi-oppositional comprehensive learning PSO [348]
Optimum routing vehicular ad-hoc networks	An opposition based ACO [119]
Truss optimization	An enhanced differential evolution (DE) algorithm with the directional mutation rule based on OBL [199]
The short-term hydrothermal scheduling problem	Quasi-reflected ions motion optimization algorithm [57]

optimization methods utilize OBL schemes just in the population level for initialization or evolutionary process with a jumping rate. Although, little efforts have been made to extend the opposition-based operations, much more effort is needed to design new opposition-based operations.

- Opposition-based multi-objective optimization: While some research works have been conducted to apply OBL scheme in the multi-objective optimization, the theoretical studies are still quite infant in this field. Therefore, it would be interesting to design the opposition-based multi-objective optimization methods which uti-

lize OBL scheme to improve distribution and accuracy of Pareto-front solutions, for instance, using Type II opposition in the Pareto frontier to obtain the better solutions and developing the opposition-based decision making (ODM) to select one solution from Pareto-front set.
- Partial opposition-based schemes: Most of opposition schemes computed the opposition for all variables in a candidate solution while it is possible to have some variables of a candidate solution which are closer to the optimal solution and applying opposition can make them worse. Only a few works have been performed [192,105] to develop partial opposition method. Therefore, a potential area is investigating new schemes of the partial opposition methods.
- Type-II opposition schemes: Developing Type-II opposition scheme is very challenging because it needs to utilize a reverse mapping, i.e., objective value to the decision vector which is a complex task especially for high-dimensional search spaces

References

[1] Point reflection, ⟨https://en.wikipedia.org/wiki/point_reflection⟩.
[2] A. Abuobieda, N. Salim, Y.J. Kumar, A.H. Osman, Opposition differential evolution based method for text summarization, in Intelligent Information and Database Systems, Springer, 2013, pp. 487–496.
[3] N. Agarwal, A.K. Dhami, J. Neogi, Synergization of different improvements in differential evolution, Mech. Eng. 1 (2013) 2.
[4] M.A. Ahandani, Opposition-based learning in the shuffled bidirectional differential evolution algorithm, Swarm Evolut. Comput. (2015).
[5] M.A. Ahandani, H. Alavi-Rad, Opposition-based learning in the shuffled differential evolution algorithm, Soft Comput. 16 (8) (2012) 1303–1337.
[6] A.R. Ajayan, S. Balaji, A modified abc algorithm & its application to wireless sensor network dynamic deployment, IOSR J. Electron. Commun. Eng. (IOSR-JECE) (2013) 2278–2834 (e-ISSN).
[7] F.S. Al-Qunaieer, H.R. Tizhoosh, and S. Rahnamayan, Opposition based computinga survey, in Neural Networks (IJCNN), The 2010 International Joint Conference on, IEEE, 2010, pp. 1–7.
[8] F.S. Al-Qunaieer, H. R. Tizhoosh, S. Rahnamayan, Oppositional fuzzy image thresholding, in Fuzzy Systems (FUZZ), 2010 IEEE International Conference on, IEEE, 2010, pp. 1–7.
[9] M. Ali, M. Pant, A. Abraham, A hybrid ant colony differential evolution and its application to water resources problems, NaBIC (2009) 1133–1138.
[10] M. Ali, P. Siarry, M. Pant, An efficient differential evolution based algorithm for solving multi-objective optimization problems, Eur. J. Oper. Res. 217 (2) (2012) 404–416.
[11] S. Allamsetty, R. Thangaraj, T.R. Chelliah, M. Pant, Sensitivity analysis on inverse characteristics of directional over current relays using differential evolution algorithm. International Journal of System Assurance Engineering and Management, pp. 1–10.
[12] Y. Ao, Differential evolution using second mutation for high-dimensional real-parameter optimization, in Measuring Technology and Mechatronics Automation in Electrical Engineering, Springer, 2012, pp. 191–201.
[13] H.V.H. Ayala, L. dos Santos Coelho, V.C. Mariani, A. Askarzadeh, An improved free search differential evolution algorithm: A case study on parameters identification of one diode equivalent circuit of a solar cell module, Energy 93 (2015) 1515–1522.
[14] H.R.L. Azad, An application of opposition based colonial competitive algorithm to solve network count location problem, Int. J. Intell. Syst. Appl. 6 (1) (2013) 29.
[15] H.R.L. Azad, N.S. Boushehri, Billboard advertising modeling by using network count location problem, Eng. J. Traffic Transp. Eng. 4 (2) (2014).
[16] H.R.L. Azad, N.S. Boushehri, N. Mollaverdi, Investigating the application of opposition concept to colonial competitive algorithm, Int. J. Bio-Inspired Comput. 4 (5) (2012) 319–329.
[17] A. Banerjee, V. Mukherjee, S. Ghoshal, Intelligent fuzzy-based reactive power compensation of an isolated hybrid power system, Int. J. Electr. Power Energy Syst. 57 (2014) 164–177.
[18] A. Banerjee, V. Mukherjee, S. Ghoshal, An opposition-based harmony search algorithm for engineering optimization problems, Ain Shams Eng. J. 5 (1) (2014) 85–101.
[19] S. Banerjee, H.R. Tizhoosh, Visualization of hidden structures in corporate failure prediction using opposite pheromone per node model, in Evolutionary Computation (CEC), 2010 IEEE Congress on, IEEE, 2010, pp. 1–5.
[20] M. Basu, Combined heat and power economic dispatch using opposition-based group search optimization, Int. J. Electr. Power Energy Syst. 73 (2015) 819–829.
[21] M. Basu, Quasi-oppositional differential evolution for optimal reactive power dispatch, Int. J. Electr. Power Energy Syst. 78 (2016) 29–40.
[22] M. Basu, Quasi-oppositional group search optimization for hydrothermal power system, Int. J. Electr. Power Energy Syst. 81 (2016) 324–335.
[23] M. Basu, Quasi-oppositional group search optimization for multi-area dynamic economic dispatch, Int. J. Electr. Power Energy Syst. 78 (2016) 356–367.
[24] A.K. Bhandari, A. Kumar, G.K. Singh, Modified artificial bee colony based computationally efficient multilevel thresholding for satellite image segmentation using kapur's, otsu and tsallis functions, Expert Syst. Appl. 42 (3) (2015) 1573–1601.
[25] K.K. Bharti, P.K. Singh, Opposition chaotic fitness mutation based adaptive inertia weight bpso for feature selection in text clustering, Appl. Soft Comput. 43 (2016) 20–34.
[26] K. Bhattacharjee, A. Bhattacharya, S.H. nee Dey, Oppositional real coded chemical reaction optimization for different economic dispatch problems, Int. J. Electr. Power Energy Syst. 55 (2014) 378–391.
[27] K.K. Bhattacharjee, S. Sarmah, A binary firefly algorithm for knapsack problems, in Industrial Engineering and Engineering Management (IEEM), 2015 IEEE International Conference on, IEEE, 2015, pp. 73–77.
[28] A. Bhattacharya, P. Chattopadhyay, Solution of economic power dispatch problems using oppositional biogeography-based optimization, Electr. Power Compon. Syst. 38 (10) (2010) 1139–1160.
[29] A. Bhattacharya, P.K. Chattopadhyay, Oppositional biogeography-based optimization for multi-objective economic emission load dispatch, in India Conference (INDICON), 2010 Annual IEEE, IEEE, 2010, pp. 1–6.
[30] A.R. Bhowmik, A.K. Chakraborty, Solution of optimal power flow using non dominated sorting multi objective opposition based gravitational search algorithm, Int. J. Electr. Power Energy Syst. 64 (2015) 1237–1250.
[31] B. Bošković, J. Brest, Tuning chess evaluation function parameters using differential evolution algorithm, Informatica 35 (2) (2011).
[32] S.M.A. Bulbul, P.K. Roy, Quasi-oppositional gravitational search algorithm applied to complex economic load dispatch problem, in Non Conventional Energy (ICONCE), 2014 Proceedings of the 1st International Conference on, IEEE, 2014, pp. 308–313.
[33] S. Cao, X. Ding, Q. Wang, B. Chen, Opposition-based improved pso for optimal reactive power dispatch and voltage control. Mathematical Problems in Engineering, 2015, 2015.
[34] Y. Cao, X. Li, J. Wang, Opposition-based animal migration optimization. Mathematical Problems in Engineering, 2013, 2013.
[35] A. Chatterjee, S. Ghoshal, V. Mukherjee, Solution of combined economic and emission dispatch problems of power systems by an opposition-based harmony search algorithm, Int. J. Electr. Power Energy Syst. 39 (1) (2012) 9–20.
[36] T.R. Chelliah, R. Thangaraj, S. Allamsetty, M. Pant, Coordination of directional overcurrent relays using opposition based chaotic differential evolution algorithm, Int. J. Electr. Power Energy Syst. 55 (2014) 341–350.
[37] C.-H. Chen, Opposition-based bare bone particle swarm optimization, in Proceedings of the 2nd International Conference on Intelligent Technologies and Engineering Systems (ICITES2013), pp. 1125–1132. Springer, 2014.
[38] C.-H. Chen, C.-M. Lin, Enhance performance of particle swarm optimization by altering the worst personal best particle, in Technologies and Applications of Artificial Intelligence (TAAI), 2012 Conference on, IEEE, 2012, pp. 56–61.
[39] K. Chen, C. Wang, Artificial bee colony algorithm improved by centroid strategy.
[40] R. Chen, J. Tang, A novel particle swarm optimisation with hybrid strategies, Int. J. Comput. Sci. Math. 6 (3) (2015) 278–286.
[41] T. Chen, Y. Wang, D. Wu, X. Wu, Z. Ma, Non-metric lens distortion correction using modified particle swarm optimisation, Int. J. Model., Identif. Control 21 (3) (2014) 330–337.
[42] Y. Chen, C. Chung, Multi-constrained optimal power flow by an opposition-based differential evolution, in Power and Energy Society General Meeting, 2012 IEEE, IEEE, 2012, pp. 1–7.
[43] M.-Y. Cheng, D.-H. Tran, Opposition-based multiple-objective differential evolution to solve the time-cost-environment impact trade-off problem in construction projects, J. Comput. Civil. Eng. 29 (5) (2014) 04014074.
[44] M.-Y. Cheng, D.-H. Tran, Integrating chaotic initialized opposition multiple-objective differential evolution and stochastic simulation to optimize ready-mixed concrete truck dispatch schedule, J. Manag. Eng. 32 (1) (2015) 04015034.
[45] Y. Chi, G. Cai, Particle swarm optimization with opposition-based disturbance, in Informatics in Control, Automation and Robotics (CAR), 2010. Proceedings of the 2nd International Asia Conference on, volume 2, IEEE, 2010, pp. 223–226.
[46] J.-P. Chiou, C.-F. Chang, A novel evolutionary algorithm for capacitor placement in distribution systems, GSTF J. Eng. Technol. (JET) 2 (3) (2013) 9.
[47] J.-P. Chiou, C.-F. Chang, J.-S. Jhang, Research for a new novel evolutionary algorithm, in Computer, Consumer and Control (IS3C), 2014 International Symposium on, IEEE, 2014, pp. 1115–1118.
[48] J. Chong, A novel multi-objective memetic algorithm based on opposition-based self-adaptive differential evolution, Memetic Comput. (2015) 1–19.
[49] J.K. Chong, X. Qiu, An opposition-based self-adaptive differential evolution with decomposition for solving the multiobjective multiple salesman problem, in Evolutionary Computation (CEC), 2016 IEEE Congress on, IEEE, 2016, pp. 4096–4103.
[50] J.K. Chong, K.C. Tan, An opposition-based self-adaptive hybridized differential evolution algorithm for multi-objective optimization (osade).In Proceedings of the 18th Asia Pacific Symposium on Intelligent and Evolutionary Systems, Volume 1, Springer, 2015, pp. 447–461.
[51] J.K. Chong, K.C. Tan, A novel grid-based differential evolution (de) algorithm for many-objective optimization, in Evolutionary Computation (CEC), 2016 IEEE Congress on, IEEE, 2016, pp. 2776–2783.
[52] L. Coelho, V.C. Mariani, H.V. Ayala, P. Alotto, Enhanced invasive weed optimization algorithm applied to electromagnetic optimization, in Proceedings of the 19th COMPUMAG Conference Comput. Electromagn. Fields, 2013, pp. 1–2.
[53] E. Cuevas, D. Oliva, D. Zaldivar, M. Perez, G. Pajares, Opposition based electromagnetismlike for global optimization. arXiv: arXiv:1405.51722014.
[54] S. Curteanu, G.D. Suditu, A.M. Buburuzan, E.N. Dragoi, Neural networks and differential evolution algorithm applied for modelling the depollution process of

some gaseous streams, Environ. Sci. Pollut. Res. 21 (22) (2014) 12856–12867.
[55] L.A. da Silveira, J. L. Soncco-Álvarez, T.A. de Lima, M. Ayala-Rincón, Memetic and opposition-based learning genetic algorithms for sorting unsigned genomes by translocations, in Advances in Nature and Biologically Inspired Computing, Springer, 2016, pp. 73–85.
[56] Y. Dai, L. Liu, S. Feng, On the identification of coupled pitch and heave motions using opposition-based particle swarm optimization. Mathematical Problems in Engineering, 2014.
[57] S. Das, A. Bhattacharya, A.K. Chakraborty, Quasi-reflected ions motion optimization algorithm for short-term hydrothermal scheduling. Neural Computing and Applications, pp. 1–27.
[58] C. Deng, X. Dong, Y. Yang, Y. Tan, X. Tan, Differential evolution with novel local search operation for large scale optimization problems, in Advances in Swarm and Computational Intelligence, Springer, 2015, pp. 317–325.
[59] H. Dhahri, A.M. Alimi, Opposition-based differential evolution for beta basis function neural network, in Evolutionary Computation (CEC), 2010 IEEE Congress on, IEEE, 2010, pp. 1–8.
[60] H. Dhahri, A.M. Alimi, Opposition-based particle swarm optimization for the design of beta basis function neural network, in Neural Networks (IJCNN), The 2010 International Joint Conference on, IEEE, 2010, pp. 1–8.
[61] H. Dhahri, A.M. Alimi, A. Abraham, Hierarchical particle swarm optimization for the design of beta basis function neural network, in Intelligent Informatics, Springer, 2013, pp. 193–205.
[62] K.K. Dhaliwal, J.S. Dhillon, Opposition aided cat swarm optimization algorithm for optimal digital iir high pass filter design.
[63] N. Dong, Y. Wang, Multiobjective differential evolution based on opposite operation, in Computational Intelligence and Security, 2009. CIS'09. International Conference on, volume 1, IEEE, 2009, pp. 123–127.
[64] N. Dong, C.-H. Wu, W.-H. Ip, Z.-Q. Chen, C.-Y. Chan, K.-L. Yung, An opposition-based chaotic ga/pso hybrid algorithm and its application in circle detection, Comput. Math. Appl. 64 (6) (2012) 1886–1902.
[65] W. Dong, L. Kang, W. Zhang, Opposition-based particle swarm optimization with adaptive mutation strategy, Soft Comput. (2016) 1–10.
[66] M. Dorigo, G. Di Caro, Ant colony optimization: a new meta-heuristic, in Evolutionary Computation, 1999. CEC 99. Proceedings of the 1999 Congress on, volume 2, IEEE, 1999, pp. 1470–1477.
[67] M. Dorigo, V. Maniezzo, A. Colorni, Ant system: optimization by a colony of cooperating agents, IEEE Trans. Syst., Man, Cybern., Part B: Cybern. 26 (1) (1996) 29–41.
[68] A. Draa, Z. Benayad, F.Z. Djenna, An opposition-based firefly algorithm for medical image contrast enhancement, Int. J. Inf. Commun. Technol. 7 (4–5) (2015) 385–405.
[69] E.-N. Dragoi, S. Curteanu, A.-I. Galaction, D. Cascaval, Optimization methodology based on neural networks and self-adaptive differential evolution algorithm applied to an aerobic fermentation process, Appl. Soft Comput. 13 (1) (2013) 222–238.
[70] M. El-Abd, Opposition-based artificial bee colony algorithm, in Proceedings of the 13th annual conference on Genetic and evolutionary computation, ACM, 2011, pp. 109–116.
[71] M. El-Abd, Generalized opposition-based artificial bee colony algorithm, in Evolutionary Computation (CEC), 2012 IEEE Congress on, IEEE, 2012, pp. 1–4.
[72] R. Ellaia, A. Habbal, E. Pagnacco, A new accelerated multi-objective particle swarm algorithm. applications to truss topology optimization, in 10th World Congress on Structural and Multidisciplinary Optimization, 2013.
[73] B.E. Emilio, E. Cuevas, et al. Optimal power flow solution using modified flower pollination algorithm, in 2015 IEEE International Autumn Meeting on Power, Electronics and Computing (ROPEC), IEEE, 2015, pp. 1–6.
[74] M. Ergezer, D. Simon, Oppositional biogeography-based optimization for combinatorial problems, in Evolutionary Computation (CEC), 2011 IEEE Congress on, IEEE, 2011, pp. 1496–1503.
[75] M. Ergezer, D. Simon, Mathematical and experimental analyses of oppositional algorithms, IEEE Trans. Cybern. 44 (11) (2014) 2178–2189.
[76] M. Ergezer, D. Simon, Probabilistic properties of fitness-based quasi-reflection in evolutionary algorithms, Comput. Oper. Res. 63 (2015) 114–124.
[77] M. Ergezer, D. Simon, D. Du, Oppositional biogeography-based optimization, SMC 9 (2009) 1009–1014.
[78] A. Eshraghi, A new approach for solving resource constrained project scheduling problems using differential evolution algorithm, Int. J. Ind. Eng. Comput. 7 (2) (2016) 205–216.
[79] A. Esmailzadeh, S. Rahnamayan, Enhanced differential evolution using center-based sampling, in Evolutionary Computation (CEC), 2011 IEEE Congress on, IEEE, 2011, pp. 2641–2648.
[80] A. Esmailzadeh, S. Rahnamayan, Center-point-based simulated annealing, in Electrical & Computer Engineering (CCECE), 2012. Proceedings of the 25th IEEE Canadian Conference on, IEEE, 2012, pp. 1–4.
[81] X. Fu, W. Liu, B. Zhang, H. Deng, Quantum behaved particle swarm optimization with neighborhood search for numerical optimization. Mathematical Problems in Engineering, 2013, 2013.
[82] H. Gao, C. Li, Opposition-based quantum firework algorithm for continuous optimisation problems, Int. J. Comput. Sci. Math. 6 (3) (2015) 256–265.
[83] Q. Gao, J. Chen, L. Wang, S. Xu, Y. Hou, Multiobjective optimization design of a fractional order pid controller for a gun control system. The Scientific World Journal, 2013, 2013.
[84] W. Gao, S. Liu, Improved artificial bee colony algorithm for global optimization, Inf. Process. Lett. 111 (17) (2011) 871–882.
[85] W. Gao, S. Liu, L. Huang, A global best artificial bee colony algorithm for global optimization, J. Comput. Appl. Math. 236 (11) (2012) 2741–2753.
[86] W.-f. Gao, S.-y. Liu, A modified artificial bee colony algorithm, Comput. Oper. Res. 39 (3) (2012) 687–697.
[87] W.-f. Gao, S.-y. Liu, L.-l. Huang, Particle swarm optimization with chaotic opposition-based population initialization and stochastic search technique, Commun. Nonlinear Sci. Numer. Simul. 17 (11) (2012) 4316–4327.
[88] X.-Z. Gao, J. Wang, J. Tanskanen, R. Bie, X. Wang, P. Guo, K. Zenger, Optimal classification of epileptic eeg signals using neural networks and harmony search methods, J. Softw. 9 (1) (2014) 230–239.
[89] X.Z. Gao, J. Wang, J.M. Tanskanen, R. Bie, P. Guo, Bp neural networks with harmony search method-based training for epileptic eeg signal classification, in Computational Intelligence and Security (CIS), 2012 Proceedings of the Eighth International Conference on, IEEE, 2012, pp. 252–257.
[90] X.Z. Gao, X. Wang, S.J. Ovaska, A hybrid harmony search method based on obl, in Computational Science and Engineering (CSE), 2010 IEEE 13th International Conference on, IEEE, 2010, pp. 140–145.
[91] X.Z. Gao, X. Wang, K. Zenger, X. Wang, A novel harmony search method with dual memory, in Systems, Man, and Cybernetics (SMC), 2012 IEEE International Conference on, IEEE, 2012, pp. 177–183.
[92] Y. Gao, X. Hu, H. Liu, F. Li, L. Peng, Opposition-based learning estimation of distribution algorithm with gaussian copulas and its application to placement of rfid readers, in Artificial Intelligence and Computational Intelligence, Springer, 2011, pp. 219–227.
[93] Y. Gao, L. Peng, F. Li, X. Hu, Velocity-free multi-objective particle swarm optimizer with centroid for wireless sensor network optimization, Artif. Intell. Comput. Intell. (2012) 682–689.
[94] Y. Gao, L. Peng, F. Li, M. Liu, W. Li, Multi-objective opposition-based learning fully informed particle swarm optimizer with favour ranking, in Granular Computing (GrC), 2013 IEEE International Conference on, IEEE, 2013, pp. 114–119.
[95] Y. Gao, L. Peng, F. Li, M. Liu, W. Liu, Opposition-based learning fully informed particle swarm optimizer without velocity, in Advances in Swarm Intelligence, Springer, 2013, pp. 79–86.
[96] Z.W. Geem, J.H. Kim, G. Loganathan, A new heuristic optimization algorithm: harmony search, Simulation 76 (2) (2001) 60–68.
[97] D.E. Goldberg, J.H. Holland, Genetic algorithms and machine learning, Mach. Learn. 3 (2) (1988) 95–99.
[98] S.K. Goudos, M. Deruyck, D. Plets, L. Martens, W. Joseph, Application of opposition-based learning concepts in reducing the power consumption in wireless access networks, in 2016 Proceedings of the 23rd International Conference on Telecommunications (ICT), IEEE, 2016, pp. 1–5.
[99] Z. Guo, H. Huang, H. Yang, S. Wang, H. Wang, An enhanced gravitational search algorithm for global optimisation, Int. J. Wirel. Mob. Comput. 9 (3) (2015) 273–280.
[100] L. Han, X. He, A novel opposition-based particle swarm optimization for noisy problems, in Natural Computation, 2007. ICNC 2007. Proceedings of the Third International Conference on, volume 3, pp. 624–629. IEEE, 2007.
[101] M. Hasan, S.N.H.S. Abdullah, Z.A. Othman, Face recognition based on opposition particle swarm optimization and support vector machine, in Signal and Image Processing Applications (ICSIPA), 2013 IEEE International Conference on, pp. 417–424. IEEE, 2013.
[102] J. He, Z. Liu, Estimation of stator resistance and rotor flux linkage in spmsm using clpso with opposition-based learning strategy.
[103] J. HU, Y. LOU, Y. CUI, Opposition-based differential evolution with ordering strategy on elite individuals?, J. Comput. Inf. Syst. 9 (23) (2013) 9421–9428.
[104] W. Hu, Y. Yu, S. Zhang, A hybrid artificial bee colony algorithm for parameter identification of uncertain fractional-order chaotic systems, Nonlinear Dyn. 82 (3) (2015) 1441–1456.
[105] Z. Hu, Y. Bao, T. Xiong, Partial opposition-based adaptive differential evolution algorithms: evaluation on the cec 2014 benchmark set for real-parameter optimization, in Evolutionary Computation (CEC), 2014 IEEE Congress on, IEEE, 2014, pp. 2259–2265.
[106] W. Huang, S.-K. Oh, Identification of fuzzy inference systems by means of a multiobjective opposition-based space search algorithm. Mathematical Problems in Engineering, 2013, 2013.
[107] G. Iacca, F. Neri, E. Mininno, Opposition-based learning in compact differential evolution, in Applications of Evolutionary Computation, Springer, 2011, pp. 264–273.
[108] M. Imran, R. Hashim, N.E.A. Khalid, Opposition based particle swarm optimization with student t mutation (ostpso), in Data Mining and Optimization (DMO), 2012. Proceedings of the 4th Conference on, IEEE, 2012, pp. 80–85.
[109] M. Imran, H. Jabeen, M. Ahmad, Q. Abbas, W. Bangyal, Opposition based pso and mutation operators, in Education Technology and Computer (ICETC), 2010. Proceedings of the 2nd International Conference on, volume 4, IEEE, 2010, pages V4–506.
[110] M.A. Iqbal, N.K. Khan, S. Akram, A.R. Baig, Hybrid mutation based evolutionary approach for function optimization, in Computer Sciences and Convergence Information Technology (ICCIT), 2011 Proceedings of the 6th International Conference on, IEEE, 2011, pages 803-808.
[111] M.A. Iqbal, N.K. Khan, M.A. Jaffar, M. Ramzan, A.R. Baig, Opposition based genetic algorithm with cauchy mutation for function optimization, in Information Science and Applications (ICISA), 2010 International Conference on, IEEE, 2010, pp. 1–7.
[112] M.A. Iqbal, N.K. Khan, H. Mujtaba, A.R. Baig, A novel function optimization approach using opposition based genetic algorithm with gene excitation, Int J. Innov. Comput. Inf. Control 7 (2011) 4263–4276.

[113] H. Jabeen, Z. Jalil, A.R. Baig, Opposition based initialization in particle swarm optimization (o-pso).In Proceedings of the 11th Annual Conference Companion on Genetic and Evolutionary Computation Conference: Late Breaking Papers, ACM, 2009, pp. 2047–2052.
[114] S. Kalra, A. Sriram, S. Rahnamayan, H. Tizhoosh, Learning opposites using neural networks, in Proceedings of the 23rd International Conference on Pattern Recognition (ICPR 2016), 2016.
[115] L. Kang, W. Dong, K. Li, Adaptive mutation opposition-based particle swarm optimization, in Computational Intelligence and Intelligent Systems, Springer, 2015, pp. 116–128.
[116] D. Karaboga, An idea based on honey bee swarm for numerical optimization. Technical report, Technical report-tr06, Erciyes university, engineering faculty, computer engineering department, 2005.
[117] K. Karthikeyan, S. Kannan, S. Baskar, C. Thangaraj, Application of opposition-based differential evolution algorithm to generation expansion planning problem, J. Electr. Eng. Technol. (JEET) 8 (4) (2013) 686–693.
[118] M. Kaucic, A multi-start opposition-based particle swarm optimization algorithm with adaptive velocity for bound constrained global optimization, J. Glob. Optim. 55 (1) (2013) 165–188.
[119] B. Kazemi, M. Ahmadi, S. Talebi, Optimum and reliable routing in vanets: An opposition based ant colony algorithm scheme, in Connected Vehicles and Expo (ICCVE), 2013 International Conference on, IEEE, 2013, pp. 926–930.
[120] B. Kazimipour, M.N. Omidvar, X. Li, A. Qin, A novel hybridization of opposition-based learning and cooperative co-evolutionary for large-scale optimization, in Evolutionary Computation (CEC), 2014 IEEE Congress on, IEEE, 2014, pp. 2833–2840.
[121] J. Kennedy, R. Eberhart, Particle swarm optimization, in Neural Networks, 1995. Proceedings., IEEE International Conference on, volume 4, IEEE, 1995, pp. 1942–1948.
[122] J. Kennedy, J.F. Kennedy, R.C. Eberhart, Y. Shi, Swarm intelligence, Morgan Kaufmann, 2001.
[123] M. Khajehzadeh, M.R. Taha, M. Eslami, Multi-objective optimisation of retaining walls using hybrid adaptive gravitational search algorithm, Civil. Eng. Environ. Syst. 31 (3) (2014) 229–242.
[124] M. Khajehzadeh, M.R. Taha, M. Eslami, Opposition-based firefly algorithm for earth slope stability evaluation, China Ocean Eng. 28 (5) (2014) 713–724.
[125] F. Khalvati, H.R. Tizhoosh, M.D. Aagaard, Opposition-based window memoization for morphological algorithms, in Computational Intelligence in Image and Signal Processing, 2007. CIISP 2007. IEEE Symposium on, IEEE, 2007, pp. 425–430.
[126] A. Khan, M.A. Jaffar, Genetic algorithm and self organizing map based fuzzy hybrid intelligent method for color image segmentation, Appl. Soft Comput. 32 (2015) 300–310.
[127] R.A. Khanum, M.A. Jan, Centroid-based initialized jade for global optimization, in Computer Science and Electronic Engineering Conference (CEEC), 2011 3rd, IEEE, 2011, pp. 115–120.
[128] J. Kofman, G.K. Knopf, Continuous unconstrained range sensing of free-form surfaces without sensor-head pose measurement, Opt. Eng. 42 (5) (2003) 1496–1510.
[129] M. Koohi-Moghadam, A.T. Rahmani, Molecular docking with opposition-based differential evolution, in Proceedings of the 27th Annual ACM Symposium on Applied Computing, ACM, 2012, pp. 1387–1392.
[130] F. Kuang, Z. Jin, W. Xu, S. Zhang, A novel chaotic artificial bee colony algorithm based on tent map, in Evolutionary Computation (CEC), 2014 IEEE Congress on, IEEE, 2014, pp. 235–241.
[131] R.M. Kumar, K. Thanushkodi, Reconfiguration and capacitor placement using opposition based differential evolution algorithm in power distribution system, Int. Rev. Model. Simul. (IREMOS) 6 (4) (2013) 1233–1239.
[132] Y. Kumar, G. Sahoo, An improved cat swarm optimization algorithm based on opposition-based learning and cauchy operator for clustering.
[133] J.-i. Kushida, A. Hara, T. Takahama, An improvement of opposition-based differential evolution with archive solutions, in Advanced Mechatronic Systems (ICAMechS), 2014 International Conference on, IEEE, 2014, pp. 463–468.
[134] L. Lei, Q. Shiru, Path planning for unmanned air vehicles using an improved artificial bee colony algorithm, in Control Conference (CCC), 2012 31st Chinese, IEEE, 2012, pp. 2486–2491.
[135] K. Lenin, B.R. Reddy, M. Suryakalavathi, Upgraded harmony search algorithm for solving optimal reactive power dispatch problem, Int. J. Math. Res. 4 (1) (2015) 42.
[136] S.W. Leung, X. Zhang, S.Y. Yuen, Multiobjective differential evolution algorithm with opposition-based parameter control, in Evolutionary Computation (CEC), 2012 IEEE Congress on, IEEE, 2012, pp. 1–8.
[137] Y.-W. Leung, Y. Wang, An orthogonal genetic algorithm with quantization for global numerical optimization, IEEE Trans. Evolut. Comput. 5 (1) (2001) 41–53.
[138] L. Li, X.F. Mai, Bacterial foraging algorithm based on quantum-behaved particle swarm optimization for global optimization, in Advanced Materials Research, volume 655, Trans Tech Publ, 2013, pp. 948–954.
[139] P. Li, W. Xu, Z. Zhou, R. Li, Optimized operation of microgrid based on gravitational search algorithm, in Electrical Machines and Systems (ICEMS), 2013 International Conference on, IEEE, 2013, pp. 338–342.
[140] X. Li, M. Yin, An opposition-based differential evolution algorithm for permutation flow shop scheduling based on diversity measure, Adv. Eng. Softw. 55 (2013) 10–31.
[141] Y. Li, X. Zhao, Y. Wang, M. Ren, Multi-objective optimization of rolling schedules for tandem hot rolling based on opposition learning multi-objective genetic algorithm, in Control and Decision Conference (CCDC), 2013 25th Chinese, IEEE, 2013, pp. 846–849.
[142] J. Lin, A hybrid discrete biogeography-based optimization for the permutation flow shop scheduling problem, Int. J. Prod. Res. (2015) 1–10.
[143] J. Lin, Oppositional backtracking search optimization algorithm for parameter identification of hyperchaotic systems, Nonlinear Dyn. 80 (1–2) (2015) 209–219.
[144] J. Lin, L. Xu, H. Zhang, Hybrid biogeography based optimization for constrained optimal spot color matching, Color Res. Appl. 39 (6) (2014) 607–615.
[145] A.W. Ling, H. Shareef, A. Mohamed, A.A. Ibrahim, An enhanced opposition-based firefly algorithm for solving complex optimization problems, J. Kejuruter. 26 (2014) 89–96.
[146] H. Liu, G. Ding, H. Sun, An improved opposition-based disruption operator in gravitational search algorithm, in Computational Intelligence and Design (ISCID), 2012 Proceedings of the Fifth International Symposium on, volume 2, IEEE, 2012, pp. 123–126.
[147] H. LIU, G. DING, G. XU, Integrating opposition-based learning into the evolutionary equation of particle swarm optimization?, J. Comput. Inf. Syst. 10 (2) (2014) 539–546.
[148] H. Liu, Z. Wu, H. Li, H. Wang, S. Rahnamayan, C. Deng, Rotation-based learning: A novel extension of opposition-based learning, in PRICAI 2014: Trends in Artificial Intelligence, Springer, 2014, pp. 511–522.
[149] H. Liu, Z. Wu, H. Wang, S. Rahnamayan, C. Deng, Improved differential evolution with adaptive opposition strategy, in Evolutionary Computation (CEC), 2014 IEEE Congress on, IEEE, 2014, pp. 1776–1783.
[150] H. Liu, G. Xu, G. Ding, D. Li, Integrating opposition-based learning into the evolution equation of bare-bones particle swarm optimization, Soft Comput. 19 (10) (2015) 2813–2836.
[151] R. Liu, L. Wang, W. Ma, C. Mu, L. Jiao, Quadratic interpolation based orthogonal learning particle swarm optimization algorithm, Nat. Comput. 13 (1) (2014) 17–37.
[152] Y. Lou, J. Li, Y. Shi, L. Jin, Gravitational co-evolution and opposition-based optimization algorithm, Int. J. Comput. Intell. Syst. 6 (5) (2013) 849–861.
[153] J. Luo, Q. Wang, A method for axis straightness error evaluation based on improved artificial bee colony algorithm, Int. J. Adv. Manuf. Technol. 71 (5–8) (2014) 1501–1509.
[154] X. Ma, F. Liu, Y. Qi, M. Gong, M. Yin, L. Li, L. Jiao, J. Wu, Moea/d with opposition-based learning for multiobjective optimization problem, Neurocomputing 146 (2014) 48–64.
[155] S. Mahdavi, S. Rahnamayan, K. Deb, Center-based initialization of cooperative co-evolutionary algorithm for large-scale optimization.
[156] M. Mahootchi, H. Tizhoosh, K. Ponnambalam, Opposition-based reinforcement learning in the management of water resources, in Approximate Dynamic Programming and Reinforcement Learning, 2007. ADPRL 2007. IEEE International Symposium on, IEEE, 2007, pp. 217–224.
[157] M. Mahootchi, H.R. Tizhoosh, K. Ponnambalam, Opposition mining in reservoir management, in Oppositional Concepts in Computational Intelligence, Springer, 2008, pp. 299–321.
[158] T. Mahto, V. Mukherjee, Energy storage systems for mitigating the variability of isolated hybrid power system, Renew. Sustain. Energy Rev. 51 (2015) 1564–1577.
[159] T. Mahto, V. Mukherjee, Evolutionary optimization technique for comparative analysis of different classical controllers for an isolated wind-diesel hybrid power system, Swarm Evolut. Comput. (2015).
[160] X. Mai, L. Ling, Pso based bacterial foraging algorithm with opposition-based learning for global optimization, Trans. Control Mech. Syst. 1 (4) (2012).
[161] K. Maji, H. Jaiswal, R. Kar, D. Mandal, S. Ghoshal, Opposition harmony search algorithm based optimal sizing of cmos analog amplifier circuit, in Science and Technology (TICST), 2015 International Conference on, IEEE, 2015, pp. 303–307.
[162] A.R. Malisia, Investigating the application of opposition-based ideas to ant algorithms. 2007.
[163] A.R. Malisia, Improving the exploration ability of ant-based algorithms, in Oppositional Concepts in Computational Intelligence, Springer, 2008, pp. 121–142.
[164] A.R. Malisia, H.R. Tizhoosh, Applying opposition-based ideas to the ant colony system, in Swarm Intelligence Symposium, 2007. SIS 2007. IEEE, IEEE, 2007, pp. 182–189.
[165] B. Mandal, P.K. Roy, Optimal reactive power dispatch using quasi-oppositional teaching learning based optimization, Int. J. Electr. Power Energy Syst. 53 (2013) 123–134.
[166] B. Mandal, T. Si, Opposition based particle swarm optimization with exploration and exploitation through gbest, in Advances in Computing, Communications and Informatics (ICACCI), 2015 International Conference on, IEEE, 2015, pp. 245–250.
[167] W. Mao, H.-y. Lan, H.-r. Li, A new modified artificial bee colony algorithm with exponential function adaptive steps, Comput. Intell. Neurosci. 501 (2015) 807630.
[168] X. Miao, D. Mu, X. Han, D. Zhang, A hybrid differential evolution for numerical optimization, in Biomedical Engineering and Informatics, 2009. BMEI'09. Proceedings of the 2nd International Conference on, IEEE, 2009, pp. 1–5.
[169] M.-E. Miranda-Varela, E. Mezura-Montes, Surrogate-assisted differential evolution with an adaptive evolution control based on feasibility to solve constrained optimization problems, in Proceedings of the Fifth International Conference on Soft Computing for Problem Solving, Springer, 2016, pp. 809–822.
[170] A. Mukherjee, V. Mukherjee, Solution of optimal power flow with facts devices using a novel oppositional krill herd algorithm, Int. J. Electr. Power Energy Syst. 78 (2016) 700–714.
[171] D.M. Muñoz, C.H. Llanos, L.D.S. Coelho, M. Ayala-Rincón, Opposition-based

shuffled pso with passive congregation applied to fm matching synthesis, in Evolutionary Computation (CEC), 2011 IEEE Congress on, IEEE, 2011, pages 2775—2781.

[172] D.M. Muñoz, C.H. Llanos, L.D.S. Coelho, M. Ayala-Rincón, Accelerating the artificial bee colony algorithm by hardware parallel implementations, in Circuits and Systems (LASCAS), 2012 IEEE Proceedings of the Third Latin American Symposium on, IEEE, 2012, pp. 1–4.

[173] D.M. Muñoz, C.H. Llanos, L.d.S. Coelho, M. Ayala-Rincón, Hardware opposition-based pso applied to mobile robot controllers, Eng. Appl. Artif. Intell. 28 (2014) 64–77.

[174] D.M. Munoz, C.H. Llanos, L. dos Santos Coelho, M. Ayala-Rincon, Hardware-based parallel firefly algorithm for embedded applications, in Adaptive Hardware and Systems (AHS), 2013 NASA/ESA Conference on, IEEE, 2013, pp. 39–46.

[175] R. MuthuKumar, K. Thanushkodi, Capacitor placement and reconfiguration of distribution system with hybrid fuzzy-opposition based differential evolution algorithm, IOSR J. Electr. Electron. Eng. (IOSR-JEEE) 6 (4) (2013) 64–69.

[176] R. Muthukumar, K. Thanushkodi, Opposition based differential evolution algorithm for capacitor placement on radial distribution system, J. Elect. Eng. Technol. 9 (2014) 45–51.

[177] R. Muthukumar, K. Thanushkodi, et al., Loss reduction in distribution system with hybrid fuzzy-opposition based differential evolution algorithm, Aust. J. Electr. Electron. Eng. 11 (2) (2014) 257.

[178] T. Niknam, M. Narimani, R. Azizipanah-Abarghooee, B. Bahmani-Firouzi, Multiobjective optimal reactive power dispatch and voltage control: a new opposition-based self-adaptive modified gravitational search algorithm, IEEE Syst. J. 7 (4) (2013) 742–753.

[179] Z. Ning, L. Ma, Z. Li, W. Xing, A hybrid particle swarm optimization for numerical optimization, in Business Intelligence and Financial Engineering, 2009. BIFE'09. International Conference on, IEEE, 2009, pp. 92–96.

[180] M.G. Omran, Using opposition-based learning with particle swarm optimization and barebones differential evolution. INTECH Open Access Publisher, 2009.

[181] M.G. Omran, Codeq: an effective metaheuristic for continuous global optimisation, Int. J. Metaheuristics 1 (2) (2010) 108–131.

[182] M.G. Omran, S. Al-Sharhan, Using opposition-based learning to improve the performance of particle swarm optimization, in Swarm Intelligence Symposium, 2008. SIS 2008. IEEE, IEEE, 2008, pp. 1–6.

[183] M.G. Omran, A.P. Engelbrecht, Free search differential evolution, in Evolutionary Computation, 2009. CEC'09. IEEE Congress on, IEEE, 2009, pages 110—117.

[184] M.G. Omran, Z.W. Geem, A. Salman, Improving the performance of harmony search using opposition-based learning and quadratic interpolation, Int. J. Math. Model. Numer. Optim. 2 (1) (2010) 28–50.

[185] H.-b. Ouyang, L.-q. Gao, X.-y. Kong, D.-x. Zou, S. Li, Teaching-learning based optimization with global crossover for global optimization problems, Appl. Math. Comput. 265 (2015) 533–556.

[186] M. Ovreiu, D. Simon, Biogeography-based optimization of neuro-fuzzy system parameters for diagnosis of cardiac disease, in Proceedings of the 12th annual conference on Genetic and evolutionary computation, ACM, 2010, pp. 1235–1242.

[187] S. ÖZYÖN, C. YAŞAR, B. DURMUŞ, H. TEMURTAŞ, Opposition-based gravitational search algorithm applied to economic power dispatch problems consisting of thermal units with emission constraints, Turk. J. Electr. Eng. Comput. Sci. 23 (2015) 2278–2288 (Sup. 1).

[188] L.D. Pacifico, T.B. Ludermir, Improved group search optimization based on opposite populations for feedforward networks training with weight decay, in Systems, Man, and Cybernetics (SMC), 2012 IEEE International Conference on, IEEE, 2012, pp. 474–479.

[189] W. Pan, K. Li, M. Wang, J. Wang, B. Jiang, Adaptive randomness: A new population initialization method. Mathematical Problems in Engineering, 2014, 2014.

[190] L. Panait, S. Luke, A comparison of two competitive fitness functions, in Genetic and Evolutionary Computation Conference (GECCO 2002) 2002, pages 503—511.

[191] S.-Y. Park, Y.-J. Kim, J.-J. Kim, J.-J. Lee, Speeded-up cuckoo search using opposition-based learning, in Control, Automation and Systems (ICCAS), 2014 14th International Conference on, IEEE, 2014, pp. 535–539.

[192] S.-Y. Park, J.-J. Lee, Stochastic opposition-based learning using a beta distribution in differential evolution. 2015.

[193] R. Patel, M. Raghuwanshi, L. Malik, A preliminary study on impact of dying of solution on performance of multi-objective genetic algorithm, in Proceedings of the Third International Conference on Soft Computing for Problem Solving, Springer, 2014, pp. 1–15.

[194] R. Patel, M. Raghuwanshi, L.G. Malik, Decomposition based multi-objective genetic algorithm (dmoga) with opposition based learning, in Computational Intelligence and Communication Networks (CICN), 2012 Proceedings of the Fourth International Conference on, IEEE, 2012, pp. 605–610.

[195] S. Paul, P.K. Roy, Optimal design of power system stabilizer using oppositional gravitational search algorithm, in Non Conventional Energy (ICONCE), 2014 1st International Conference on, IEEE, 2014, pp. 282–287.

[196] Y. Pei, H. Takagi, Fitness landscape approximation by adaptive support vector regression with opposition-based learning, in Systems, Man, and Cybernetics (SMC), 2013 IEEE International Conference on, IEEE, 2013, pp. 1329–1334.

[197] Y. Pei, H. Takagi, Triple and quadruple comparison-based interactive differential evolution and differential evolution, in Proceedings of the twelfth workshop on Foundations of genetic algorithms XII, ACM, 2013, pp. 173–182.

[198] L. Peng, Y. Wang, G. Dai, A novel opposition-based multi-objective differential evolution algorithm for multi-objective optimization, in Advances in Computation and Intelligence, Springer, 2008, pp. 162–170.

[199] A.H. Pham, Discrete optimal sizing of truss using adaptive directional differential evolution, Adv. Comput. Des. 1 (3) (2016) 275–296.

[200] R. Poláková, J. Tvrdík, Various mutation strategies in enhanced competitive differential evolution for constrained optimization, in Differential Evolution (SDE), 2011 IEEE Symposium on, IEEE, 2011, pp. 1–8.

[201] F.M. Pour, A.A. Gharaveisi, Opposition-based discrete action reinforcement learning automata algorithm case study: optimal design of a pid controller, Turk. J. Electr. Eng. Comput. Sci. 21 (6) (2013) 1603–1614.

[202] K.V. Price, An introduction to differential evolution. 1999.

[203] A. Qin, F. Forbe, Dynamic regional harmony search algorithm with opposition and local learning.

[204] A.R. Rahman, B. Santosa, S.E. Wiratno, Hybrid differential evolution and bottleneck heuristic algorithm to solve bi-objective hybrid flow shop scheduling unrelated parallel machines problem, in International Conference on Industrial Engineering and Operations Management, pp. 1339–1347.

[205] S. Rahnamayan, Opposition-based differential evolution. 2007.

[206] S. Rahnamayan, J. Jesuthasan, F. Bourennani, H. Salehinejad, G.F. Naterer, Computing opposition by involving entire population, in Evolutionary Computation (CEC), 2014 IEEE Congress on, pp. 1800–1807. IEEE, 2014.

[207] S. Rahnamayan, H.R. Tizhoosh, Image thresholding using micro opposition-based differential evolution (micro-ode), in Evolutionary Computation, 2008. CEC 2008. (IEEE World Congress on Computational Intelligence). IEEE Congress on, pp. 1409–1416. IEEE, 2008.

[208] S. Rahnamayan, H.R. Tizhoosh, M. Salama, Opposition-based differential evolution algorithms, in Evolutionary Computation, 2006. CEC 2006. IEEE Congress on, pp. 2010–2017. IEEE.

[209] S. Rahnamayan, H. R. Tizhoosh, M. Salama, Quasi-oppositional differential evolution. in Evolutionary Computation, 2007. CEC 2007. IEEE Congress on, pp. 2229–2236. IEEE, 2007.

[210] S. Rahnamayan, H.R. Tizhoosh, M. Salama, Opposition-based differential evolution, IEEE Trans. Evolut. Comput. 12 (1) (2008) 64–79.

[211] S. Rahnamayan G.G. Wang, Investigating in scalability of opposition-based differential evolution, in Proceedings of the 8th WSEAS International Conference on Simulation, Modeling and Optimization (SMO08), Santander, Cantabria, Spain, pp. 105–111, 2008.

[212] S. Rahnamayan, G.G. Wang, Solving large scale optimization problems by opposition-based differential evolution (ode), WSEAS Trans. Comput. 7 (10) (2008) 1792–1804.

[213] S. Rahnamayan, G.G. Wang, Center-based initialization for large-scale black-box problems, in Proceedings of the 8th WSEAS international conference on Artificial intelligence, knowledge engineering and data bases, pp. 531–541. World Scientific and Engineering Academy and Society (WSEAS), 2009.

[214] S. Rahnamayan, G.G. Wang, Center-based sampling for population-based algorithms, in Evolutionary Computation, 2009. CEC'09. IEEE Congress on, pp. 933–938. IEEE, 2009.

[215] S. Rahnamayan, G.G. Wang, Toward effective initialization for large-scale search spaces, Trans. Syst. 8 (3) (2009) 355–367.

[216] S. Rahnamayan, G.G. Wang, M. Ventresca, An intuitive distance-based explanation of opposition-based sampling, Appl. Soft Comput. 12 (9) (2012) 2828–2839.

[217] A. Rajasekhar, S. Das, Teaching learning opposition based optimization for the location of median line in 3-d space, in Swarm, Evolutionary, and Memetic Computing, pp. 331–338. Springer, 2012.

[218] A. Rajasekhar, R.K. Jatoth, A. Abraham, Design of intelligent pid/pi λ d μ speed controller for chopper fed dc motor drive using opposition based artificial bee colony algorithm, Eng. Appl. Artif. Intell. 29 (2014) 13–32.

[219] A. Rajasekhar, R. Rani, K. Ramya, A. Abraham, Elitist teaching learning opposition based algorithm for global optimization, in Systems, Man, and Cybernetics (SMC), 2012 IEEE International Conference on, pp. 1124–1129. IEEE, 2012.

[220] G. Ram, D. Mandal, R. Kar, S.P. Ghoshal, Opposition-based bat algorithm for optimal design of circular and concentric circular arrays with improved far-field radiation characteristics, International Journal of Numerical Modelling: Electronic Networks, Devices and Fields, 2015.

[221] A. Rashid, B. Kim, A. Khambampati, S. Kim, K. Kim, An oppositional biogeography-based optimization technique to reconstruct organ boundaries in the human thorax using electrical impedance tomography, Physiol. Meas. 32 (7) (2011) 767.

[222] M. Rashid, A.R. Baig, Improved opposition-based pso for feedforward neural network training, in Information Science and Applications (ICISA), 2010 International Conference on, pp. 1–6. IEEE, 2010.

[223] S. Reghunathan, T. Baby, Performance evaluation of opposition based differential evolution on non-convex economic dispatch, in Advances in Power Conversion and Energy Technologies (APCET), 2012 International Conference on, pp. 1–6. IEEE, 2012.

[224] M. Remli, S.b. Deris, M. Jamous, M.S. Mohamad, A. Abdullah, Service composition optimization using differential evolution and opposition-based learning. 2015.

[225] N. Rojas-Morales, M.-C. Riff, E. Montero, Ants can learn from the opposite, in Proceedings of the 2016 on Genetic and Evolutionary Computation Conference, pp. 389–396. ACM, 2016.

[226] N. Rojas-Morales, M.-C. Riff, E. Montero, Learning from the opposite: Strategies for ants that solve multidimensional knapsack problem, in Evolutionary Computation (CEC), 2016 IEEE Congress on, pp. 193–200. IEEE, 2016.

[227] C.D. Rosin, R.K. Belew, New methods for competitive coevolution, Evolut. Comput. 5 (1) (1997) 1–29.

[228] A. Roy, S. Dutta, P.K. Roy, Automatic generation control by smes-smes controllers

of two-area hydro-hydro system, in Non Conventional Energy (ICONCE), 2014 1st International Conference on, pages 302-307. IEEE, 2014.
[229] P. Roy, D. Mandal, Quasi-oppositional biogeography-based optimization for multi-objective optimal power flow, Electr. Power Compon. Syst. 40 (2) (2011) 236–256.
[230] P.K. Roy, S. Bhui, Multi-objective quasi-oppositional teaching learning based optimization for economic emission load dispatch problem, Int. J. Electr. Power Energy Syst. 53 (2013) 937–948.
[231] P.K. Roy, D. Mandal, Optimal reactive power dispatch using quasi-oppositional biogeography-based optimization, Int. J. Energy Optim. Eng. (IJEOE) 1 (4) (2012) 38–55.
[232] P.K. Roy, C. Paul, Quasi-oppositional gravitational search algorithm applied to short term hydrothermal scheduling problems, Int. J. Power Energy Convers. 6 (2) (2015) 165–185.
[233] P.K. Roy, C. Paul, S. Sultana, Oppositional teaching learning based optimization approach for combined heat and power dispatch, Int. J. Electr. Power Energy Syst. 57 (2014) 392–403.
[234] P.K. Roy, R. Sarkar, Solution of unit commitment problem using quasi-oppositional teaching learning based algorithm, Int. J. Electr. Power Energy Syst. 60 (2014) 96–106.
[235] P.K. Roy, A. Sur, D.K. Pradhan, Optimal short-term hydro-thermal scheduling using quasi-oppositional teaching learning based optimization, Eng. Appl. Artif. Intell. 26 (10) (2013) 2516–2524.
[236] W.F. Sacco, A.C. Rios-Coelho, N. Henderson, Testing population initialisation schemes for differential evolution applied to a nuclear reactor core design, Int. J. Nucl. Energy Sci. Technol. 8 (3) (2014) 192–212.
[237] S.K. Saha, R. Kar, D. Mandal, S.P. Ghoshal, V. Mukherjee, A new design method using opposition-based bat algorithm for iir system identification problem, Int. J. Bio-Inspired Comput. 5 (2) (2013) 99–132.
[238] F. Sahba, H.R. Tizhoosh, Opposite actions in reinforced image segmentation, in Oppositional Concepts in Computational Intelligence, pp. 287–297. Springer, 2008.
[239] F. Sahba, H.R. Tizhoosh, M.M. Salama, Application of opposition-based reinforcement learning in image segmentation, in Computational Intelligence in Image and Signal Processing, 2007. CIISP 2007. IEEE Symposium on, pp. 246–251. IEEE, 2007.
[240] F. Saki, A. Tahmasbi, S.B. Shokouhi, A novel opposition-based classifier for mass diagnosis in mammography images, in Biomedical Engineering (ICBME), 2010 17th Iranian Conference of, pages 1-4. IEEE, 2010.
[241] F. Saki, A. Tahmasbi, H. Soltanian-Zadeh, S.B. Shokouhi, Fast opposite weight learning rules with application in breast cancer diagnosis, Comput. Biol. Med. 43 (1) (2013) 32–41.
[242] H. Salehinejad, S. Rahnamayan, H.R. Tizhoosh, Type-ii opposition-based differential evolution, in Evolutionary Computation (CEC), 2014 IEEE Congress on, pp. 1768–1775. IEEE, 2014.
[243] G. Samanta, A. Chandra, A novel design strategy of low-pass fir filter using opposition-based differential evolution algorithm, in Electrical, Electronics and Computer Science (SCEECS), 2012 IEEE Students' Conference on, pp. 1–4. IEEE, 2012.
[244] Z. Seif, M.B. Ahmadi, An opposition-based algorithm for function optimization, Eng. Appl. Artif. Intell. 37 (2015) 293–306.
[245] Z. Seif, M.B. Ahmadi, Opposition versus randomness in binary spaces, Appl. Soft Comput. 27 (2015) 28–37.
[246] F. Shahzad, A.R. Baig, S. Masood, M. Kamran, N. Naveed, Opposition-based particle swarm optimization with velocity clamping (ovcpso), in Advances in Computational Intelligence, pp. 339–348. Springer, 2009.
[247] F. Shahzad, S. Masood, N.K. Khan, Probabilistic opposition-based particle swarm optimization with velocity clamping, Knowl. Inf. Syst. 39 (3) (2014) 703–737.
[248] G. Shankar, V. Mukherjee, Quasi oppositional harmony search algorithm based controller tuning for load frequency control of multi-source multi-area power system, Int. J. Electr. Power Energy Syst. 75 (2016) 289–302.
[249] H. Sharma, J.C. Bansal, K. Arya, Opposition based lévy flight artificial bee colony, Memetic Comput. 5 (3) (2013) 213–227.
[250] T.K. Sharma, M. Pant, Intermediate population based differential evolution algorithm, in Computational Intelligence and Information Technology, pp. 152–156. Springer, 2011.
[251] B. Shaw, V. Mukherjee, S. Ghoshal, A novel opposition-based gravitational search algorithm for combined economic and emission dispatch problems of power systems, Int. J. Electr. Power Energy Syst. 35 (1) (2012) 21–33.
[252] B. Shaw, V. Mukherjee, S. Ghoshal, Solution of reactive power dispatch of power systems by an opposition-based gravitational search algorithm, Int. J. Electr. Power Energy Syst. 55 (2014) 29–40.
[253] B. Shaw, V. Mukherjee, S.P. Ghoshal, Solution of optimal reactive power dispatch by an opposition-based gravitational search algorithm, in Swarm, Evolutionary, and Memetic Computing, pp. 558–567. Springer, 2013.
[254] C.K. Shiva, V. Mukherjee, Automatic generation control of interconnected power system for robust decentralized random load disturbances using a novel quasi-oppositional harmony search algorithm, Int. J. Electr. Power Energy Syst. 73 (2015) 991–1001.
[255] C.K. Shiva, V. Mukherjee, Comparative performance assessment of a novel quasi-oppositional harmony search algorithm and internal model control method for automatic generation control of power systems, Gener., Transm. Distrib., IET 9 (11) (2015) 1137–1150.
[256] C.K. Shiva, V. Mukherjee, A novel quasi-oppositional harmony search algorithm for automatic generation control of power system, Appl. Soft Comput. 35 (2015) 749–765.
[257] C.K. Shiva, V. Mukherjee, Automatic generation control of hydropower systems using a novel quasi-oppositional harmony search algorithm, Electr. Power Compon. Syst. 44 (13) (2016) 1478–1491.
[258] M. Shokri, Knowledge of opposite actions for reinforcement learning, Appl. Soft Comput. 11 (6) (2011) 4097–4109.
[259] M. Shokri, H.R. Tizhoosh, M. Kamel, Opposition-based q (λ) algorithm, in Neural Networks, 2006. IJCNN'06. International Joint Conference on, pp. 254–261. IEEE, 2006.
[260] M. Shokri, H.R. Tizhoosh, M.S. Kamel, Opposition-based q (λ) with non-markovian update, in Approximate Dynamic Programming and Reinforcement Learning, 2007. ADPRL 2007. IEEE International Symposium on, pp. 288–295. IEEE, 2007.
[261] M. Shokri, H.R. Tizhoosh, M.S. Kamel, The concept of opposition and its use in q-learning and q (λ) techniques. in Oppositional Concepts in Computational Intelligence, pp. 233–253. Springer, 2008.
[262] M. Shokri, H.R. Tizhoosh, M.S. Kamel, Tradeoff between exploration and exploitation of oq (λ) with non-markovian update in dynamic environments, in Neural Networks, 2008. IJCNN 2008.(IEEE World Congress on Computational Intelligence). IEEE International Joint Conference on, pp. 2915–2921. IEEE, 2008.
[263] M. Shokri, H.R. Tizhoosh, M.S. Kamel, Oppositional target domain estimation using grid-based simulation, Appl. Soft Comput. 9 (1) (2009) 423–430.
[264] D. Sidhu, J. Dhillon, D. Kaur, Hybrid heuristic search method for design of digital iir filter with conflicting objectives, Soft Comput. (2016) 1–16.
[265] D. Sidhu, J.S. Dhillon, D. Kaur, Design of higher order digital iir low pass filter using hybrid differential evolution, memory 12 (2016) 21.
[266] A. Silva, A. Neves, T. Gonçalves, Using scout particles to improve a predator-prey optimizer, in Adaptive and Natural Computing Algorithms, pp. 130–139. Springer, 2013.
[267] D. Simon, Biogeography-based optimization, IEEE Trans. Evolut. Comput. 12 (6) (2008) 702–713.
[268] B. Singh, J.S. Dhillon, Y. Brar, A hybrid differential evolution method for the design of iir digital filter, ACEEE Int. J. Signal Image Process. 4 (1) (2013) 1–10.
[269] B.J. Singh, J.S. Dhillon, Higher order optimal stable digital iir filter design using heuristic optimization, in Proceedings of Informing Science & IT Education Conference (InSITE) 2015, pages 505-520, 2015.
[270] D. SINGH, J.S. DHILLON, Fuzzy based design of digital iir filter using etlbo.
[271] D. Singh, J.S. Dhillon, Teaching-learning based optimization technique for the design of lp and hp digital iir filter, Recent Adv. Electr. Eng. Electron. Devices (2013) 203–208.
[272] D. Singh, R. Kaur, Hybrid optimization technique for the design of digital differentiator, Recent Res. Circuits, Syst., Mech. Transp. Syst. (2011) 52–57.
[273] R. Singh, V. Mukherjee, S. Ghoshal, The opposition-based harmony search algorithm, J. Inst. Eng. (India): Ser. B 94 (4) (2013) 247–256.
[274] R. Storn, K. Price, Differential evolution-a simple and efficient heuristic for global optimization over continuous spaces, J. Glob. Optim. 11 (4) (1997) 341–359.
[275] B. Subudhi, D. Jena, Nonlinear system identification using opposition based learning differential evolution and neural network techniques. 2009.
[276] B. Subudhi, D. Jena, A differential evolution based neural network approach to nonlinear system identification, Appl. Soft Comput. 11 (1) (2011) 861–871.
[277] S. Sultana, P.K. Roy, Multi-objective quasi-oppositional teaching learning based optimization for optimal location of distributed generator in radial distribution systems, Int. J. Electr. Power Energy Syst. 63 (2014) 534–545.
[278] S. Sultana, P.K. Roy, Oppositional krill herd algorithm for optimal location of capacitor with reconfiguration in radial distribution system, Int. J. Electr. Power Energy Syst. 74 (2016) 78–90.
[279] P. Surekha, D.S. Sumathi, Solving economic load dispatch problems using differential evolution with opposition based learning, Wseas Trans. Inf. Sci. Appl. 1 (2012).
[280] P. Surekha, S. Sumathi, An improved differential evolution algorithm for optimal load dispatch in power systems including transmission losses, IU- J. Electr. Electron. Eng. 11 (2) (2011) 1379–1390.
[281] A. Tahmasbi, F. Saki, S.B. Shokouhi, Classification of benign and malignant masses based on zernike moments, Comput. Biol. Med. 41 (8) (2011) 726–735.
[282] T. Takagi, M. Sugeno, Fuzzy identification of systems and its applications to modeling and control, IEEE Trans. Syst., Man Cybern. 1 (1985) 116–132.
[283] A. Talukder, K. Deb, S. Rahnamayan, Maintaining diversity in the bounded pareto-set: A case of opposition based solution generation scheme, in Proceedings of the 2016 on Genetic and Evolutionary Computation Conference Companion, pp. 945–951. ACM, 2016.
[284] T.G. Tan, J. Teo, Evolving opposition-based pareto solutions: Multiobjective optimization using competitive coevolution, in Oppositional Concepts in Computational Intelligence, pp. 161–206. Springer, 2008.
[285] J. Tang, X. Zhao, An enhanced opposition-based particle swarm optimization, in Intelligent Systems, 2009. GCIS'09. WRI Global Congress on, volume 1, pp. 149–153. IEEE, 2009.
[286] J. Tang, X. Zhao, On the improvement of opposition-based differential evolution, in Natural Computation (ICNC), 2010 Sixth International Conference on, volume 5, pp. 2407–2411. IEEE, 2010.
[287] J. Tao, A particle swarm optimization algorithm for neural networks in recognition of maize leaf diseases, in Computer and Computing Technologies in Agriculture VIII, pp. 495–505. Springer, 2014.
[288] M. Tarkeshwar, V. Mukherjee, A novel quasi-oppositional harmony search algorithm and fuzzy logic controller for frequency stabilization of an isolated hybrid power system, Int. J. Electr. Power Energy Syst. 66 (2015) 247–261.
[289] M. Tarkeshwar, V. Mukherjee, Quasi-oppositional harmony search algorithm and

fuzzy logic controller for load frequency stabilisation of an isolated hybrid power system, Gener., Transm. Distrib., IET 9 (5) (2015) 427–444.
[290] M.F. Tasgetiren, P. Suganthan, S. Ozcan, D. Kizilay, A differential evolution algorithm with a variable neighborhood search for constrained function optimization, in Adaptation and hybridization in computational intelligence, pp. 171–184. Springer, 2015.
[291] R. Thangaraj, M. Pant, Differential evolution algorithm for solving multi-objective optimization problems, Proceedings: Recent Advances in Mathematics, Cambridge, MA, USA, pp. 40–45, 2013.
[292] R. Thangaraj, M. Pant, T.R. Chelliah, A. Abraham, Opposition based chaotic differential evolution algorithm for solving global optimization problems, in Nature and Biologically Inspired Computing (NaBIC), 2012 Fourth World Congress on, pp. 1–7. IEEE, 2012.
[293] B.S. Theja, A. Rajasekhar, A. Abraham, An optimal design of coordinated pi based pss with tcsc controller using modified teaching learning based optimization, in Nature and Biologically Inspired Computing (NaBIC), 2013 World Congress on, pages 99-106. IEEE, 2013.
[294] K. Thenmalar, S. Ramesh, Hybrid fuzzy-opposition based differential evolution algorithm (fodea) for dynamic economic emission power dispatch (eepd) with emission constraints and valve point effects, Middle-East J. Sci. Res. 23 (10) (2015) 2507–2520.
[295] K. Thenmalar, S. Ramesh, S. Thiruvenkadam, Opposition based differential evolution algorithm for dynamic economic emission load dispatch (eeld) with emission constraints and valve point effects, J. Electr. Eng. Technol. 10 (4) (2015) 1508–1517.
[296] H. Tizhoosh, M. Ventresca, Oppositional concepts in computational intelligence, ser. Studies in Computational Intelligence. Berlin, Germany: Springer, 155, 2008.
[297] H.R. Tizhoosh, Opposition-based learning: a new scheme for machine intelligence, in null, pages 695-701. IEEE, 2005.
[298] H.R. Tizhoosh, Reinforcement learning based on actions and opposite actions, in International conference on artificial intelligence and machine learning, volume 414, 2005.
[299] H.R. Tizhoosh, Opposite fuzzy sets with applications in image processing, in IFSA/EUSFLAT Conference, pp. 36–41, 2009.
[300] H.R. Tizhoosh, S. Rahnamayan, Learning opposites with evolving rules, in Fuzzy Systems (FUZZ-IEEE), 2015 IEEE International Conference on, pages 1-8. IEEE, 2015.
[301] H.R. Tizhoosh, F. Sahba, Quasi-global oppositional fuzzy thresholding, in Fuzzy Systems, 2009. FUZZ-IEEE 2009. IEEE International Conference on, pp. 1346–1351. IEEE, 2009.
[302] H.R. Tizhoosh, M. Ventresca, S. Rahnamayan, Opposition-based computing. in Oppositional Concepts in Computational Intelligence, pp. 11–28. Springer, 2008.
[303] O.E. Turgut, M.S. Turgut, M.T. Coban, Design and economic investigation of shell and tube heat exchangers using improved intelligent tuned harmony search algorithm, Ain Shams Eng. J. 5 (4) (2014) 1215–1231.
[304] J. Tvrdík, R. Poláková, Enhanced competitive differential evolution for constrained optimization, in Computer Science and Information Technology (IMCSIT), Proceedings of the 2010 International Multiconference on, pages 909-915. IEEE, 2010.
[305] J. Tvrdík, R. Poláková, J. Veselsky`, P. Bujok, Adaptive variants of differential evolution: Towards control-parameter-free optimizers, in Handbook of Optimization, pp. 423–449. Springer, 2013.
[306] P. Upadhyay, R. Kar, D. Mandal, S. Ghoshal, V. Mukherjee, A novel design method for optimal iir system identification using opposition based harmony search algorithm, J. Frankl. Inst. 351 (5) (2014) 2454–2488.
[307] M. Ventresca, H.R. Tizhoosh, Improving the convergence of backpropagation by opposite transfer functions, in Neural Networks, 2006. IJCNN'06. International Joint Conference on, pp. 4777–4784. IEEE, 2006.
[308] M. Ventresca, H.R. Tizhoosh, Opposite transfer functions and backpropagation through time, in Foundations of Computational Intelligence, 2007. FOCI 2007. IEEE Symposium on, pages 570-577. IEEE, 2007.
[309] M. Ventresca, H.R. Tizhoosh, Simulated annealing with opposite neighbors. in Foundations of Computational Intelligence, 2007. FOCI 2007. IEEE Symposium on, pages 186-192. IEEE, 2007.
[310] M. Ventresca, H.R. Tizhoosh, A diversity maintaining population-based incremental learning algorithm, Inf. Sci. 178 (21) (2008) 4038–4056.
[311] M. Ventresca, H.R. Tizhoosh, Numerical condition of feedforward networks with opposite transfer functions, in Neural Networks, 2008. IJCNN 2008.(IEEE World Congress on Computational Intelligence).IEEE International Joint Conference on, pp. 3233–3240. IEEE, 2008.
[312] M. Ventresca, H.R. Tizhoosh, Two frameworks for improving gradient-based learning algorithms, in Oppositional Concepts in Computational Intelligence, pp. 255–284. Springer, 2008.
[313] M. Ventresca, H.R. Tizhoosh, Improving gradient-based learning algorithms for large scale feedforward networks, in Neural Networks, 2009. IJCNN 2009. International Joint Conference on, pp. 3212–3219. IEEE, 2009.
[314] O.P. Verma, D. Aggarwal, T. Patodi, Opposition and dimensional based modified firefly algorithm, Expert Syst. Appl. 44 (2016) 168–176.
[315] A. Wang, X.S. He, F. Wang, Opposition-based particle swarm optimization with plow operator, in Cross Strait Quad-Regional Radio Science and Wireless Technology Conference (CSQRWC), 2011, volume 2, pp. 1696–1699. IEEE, 2011.
[316] B. Wang, A novel artificial bee colony algorithm based on modified search strategy and generalized opposition-based learning, J. Intell. Fuzzy Syst. 28 (3) (2015) 1023–1037.
[317] D. Wang, C. Xiong, X. Zhang, An opposition-based group search optimizer with diversity guidance, Mathematical Problems in Engineering, 2015, 2015.
[318] H. Wang, Opposition-based barebones particle swarm for constrained nonlinear optimization problems, Mathematical Problems in Engineering, 2012, 2012.
[319] H. Wang, H. Li, Y. Liu, H. Li, S. Zeng, Opposition-based particle swarm algorithm with cauchy mutation, in Evolutionary Computation, 2007. CEC 2007. IEEE Congress on, pp. 4750–4756. IEEE, 2007.
[320] H. Wang, L. Liang, Z. Niu, Z. He, Identification of ctqs for complex products based on mutual information and improved gravitational search algorithm, Mathematical Problems in Engineering, 2015, 2015.
[321] H. Wang, H. Ouyang, L. Gao, W. Qin, Opposition-based learning harmony search algorithm with mutation for solving global optimization problems, in Control and Decision Conference (2014 CCDC), The 26th Chinese, pp. 1090–1094. IEEE, 2014.
[322] H. Wang, S. Rahnamayan, Z. Wu, Parallel differential evolution with self-adapting control parameters and generalized opposition-based learning for solving high-dimensional optimization problems, J. Parallel Distrib. Comput. 73 (1) (2013) 62–73.
[323] H. Wang, S. Rahnamayan, S. Zeng, Generalised opposition-based differential evolution: an experimental study, Int. J. Comput. Appl. Technol. 43 (4) (2012) 311–319.
[324] H. Wang, H. Sun, C. Li, S. Rahnamayan, J.-S. Pan, Diversity enhanced particle swarm optimization with neighborhood search, Inf. Sci. 223 (2013) 119–135.
[325] H. Wang, W. Wang, H. Sun, Firefly algorithm with generalised opposition-based learning, Int. J. Wirel. Mob. Comput. 9 (4) (2015) 370–376.
[326] H. Wang, W. Wang, H. Sun, Z. Cui, S. Rahnamayan, S. Zeng, A new cuckoo search algorithm with hybrid strategies for flow shop scheduling problems, Soft Comput. (2016) 1–11.
[327] H. Wang, W. Wang, H. Sun, S. Rahnamayan, Using opposition-based learning to enhance differential evolution: A comparative study, in Electrical & Computer Engineering (CCECE), 2016 25th IEEE Canadian Conference on. IEEE, 2016.
[328] H. Wang, Z. Wu, Y. Liu, J. Wang, D. Jiang, L. Chen, Space transformation search: a new evolutionary technique, in Proceedings of the first ACM/SIGEVO Summit on Genetic and Evolutionary Computation, pages 537-544. ACM, 2009.
[329] H. Wang, Z. Wu, S. Rahnamayan, Enhanced opposition-based differential evolution for solving high-dimensional continuous optimization problems, Soft Comput. 15 (11) (2011) 2127–2140.
[330] H. Wang, Z. Wu, S. Rahnamayan, L. Kang, A scalability test for accelerated de using generalized opposition-based learning, in Intelligent Systems Design and Applications, 2009. ISDA'09. Proceedings of the Ninth International Conference on, pp. 1090–1095. IEEE, 2009.
[331] H. Wang, Z. Wu, S. Rahnamayan, J. Wang, Diversity analysis of opposition-based differential evolutionan experimental study, in Advances in Computation and Intelligence, pp. 95–102. Springer, 2010.
[332] H. Wang, Z. Wu, J. Wang, X. Dong, S. Yu, C. Chen, A new population initialization method based on space transformation search, in Natural Computation, 2009. ICNC'09. Fifth International Conference on, volume 5, pages 332-336. IEEE, 2009.
[333] J. Wang, Particle swarm optimization with adaptive parameter control and opposition, J. Comput. Inf. Syst. 7 (12) (2011) 4463–4470.
[334] J. Wang, An enhanced differential evolution algorithm for solving large scale optimisation problems on graphics hardware, Int. J. Comput. Appl. Technol. 46 (3) (2013) 259–266.
[335] J. Wang, Enhanced differential evolution with generalised opposition-based learning and orientation neighbourhood mining, Int. J. Comput. Sci. Math. 6 (1) (2015) 49–58.
[336] J. Wang, Z. Wu, H. Wang, Hybrid differential evolution algorithm with chaos and generalized opposition-based learning, in Advances in Computation and Intelligence, pp. 103–111. Springer, 2010.
[337] J. Wang, Z. Wu, H. Wang, A novel particle swarm algorithm for solving parameter identification problems on graphics hardware, Int. J. Comput. Sci. Eng. 6 (1–2) (2011) 43–51.
[338] K. Wang, W. Ma, H. Luo, H. Qin, Coordinated scheduling of production and transportation in a two-stage assembly flowshop, Int. J. Prod. Res. (2016) 1–21.
[339] W. Wang, H. Wang, S. Rahnamayan, Improving comprehensive learning particle swarm optimiser using generalised opposition-based learning, Int. J. Model., Identif. Control 14 (4) (2011) 310–316.
[340] Y. Wang, Z. Cai, Q. Zhang, Enhancing the search ability of differential evolution through orthogonal crossover, Inf. Sci. 185 (1) (2012) 153–177.
[341] W. Wei, J. Zhou, F. Chen, H. Yuan, Constrained differential evolution using generalized opposition-based learning, Soft Comput. (2016) 1–25.
[342] Y. Wen, L. Liu, Z. Wang, J. Kou, Multi-ucavs targets assignment using opposition-based genetic algorithm, in Control and Decision Conference (CCDC), 2015 27th Chinese, pp. 6026–6030. IEEE, 2015.
[343] L.A. Wong, H. Shareef, A. Mohamed, A.A. Ibrahim, An enhanced opposition-based firefl y algorithm for solving complex optimization problems (algoritma kelip berasaskan teori tentangan untuk penyelesaian masalah pengoptimuman kompleks), J. Kejuruter. (J. Eng.) 26 (2014) 89–96.
[344] L.A. Wong, H. Shareef, A. Mohamed, A.A. Ibrahim, Optimal battery sizing in photovoltaic based distributed generation using enhanced opposition-based firefly algorithm for voltage rise mitigation. The Scientific World Journal, 2014, 2014.
[345] X. Wu, J. Kofman, H.R. Tizhoosh, Active exploratory q-learning for large problems, in Systems, Man and Cybernetics, 2007. ISIC. IEEE International Conference on, pp. 4040–4045. IEEE, 2007.
[346] Y. Wu, B. Zhao, J. Guo, A fast opposition-based differential evolution with cauchy mutation, in Intelligent Systems (GCIS), 2012 Third Global Congress on, pages 72-75. IEEE, 2012.
[347] Z. Wu, Z. Ni, C. Zhang, L. Gu, Opposition based comprehensive learning particle

swarm optimization, in Intelligent System and Knowledge Engineering, 2008. ISKE 2008. 3rd International Conference on, volume 1, pp. 1013–1019. IEEE, 2008.

[348] Z. Wu, Z. Ni, C. Zhang, L. Gu, A novel pso for multi-stage portfolio planning, in 2009 International Conference on Artificial Intelligence and Computational Intelligence, pages 71-77. IEEE, 2009.

[349] W.-l. Xiang, M.-q. An, Y.-z. Li, R.-c. He, J.-f. Zhang, An improved global-best harmony search algorithm for faster optimization, Expert Syst. Appl. 41 (13) (2014) 5788–5803.

[350] W.-l. Xiang, X.-l. Meng, M.-q. An, Y.-z. Li, M.-x. Gao, An enhanced differential evolution algorithm based on multiple mutation strategies. Computational intelligence and neuroscience, 2015, 2015.

[351] C. Xie, W. Chen, W. Yu, A hybrid group search optimizer with opposition-based learning and differential evolution, in Computational Intelligence and Intelligent Systems, pp. 3–12. Springer, 2015.

[352] J. Xie, J. Yang, Improved differential evolution for global optimization, in Information Management and Engineering (ICIME), 2010 The 2nd IEEE International Conference on, pp. 651–654. IEEE, 2010.

[353] M. Xiongfa, L. Ling, Bacterial foraging algorithm based on quantum-behaved particle swarm optimization and opposition-based learning, J. Comput. Inf. Syst. 3 (2013) 1157–1165.

[354] D. Xu, X. Ai, An improved diversity guided particle swarm optimization, in Proceedings of the Sixth International Symposium on Neural Networks (ISNN 2009), pages 623-630. Springer, 2009.

[355] H. Xu, C.D. Erdbrink, V.V. Krzhizhanovskaya, How to speed up optimization? Opposite-center learning and its application to differential evolution, Procedia Comput. Sci. 51 (2015) 805–814.

[356] Q. Xu, L. Guo, N. Wang, Y. He, Coobbo: a novel opposition-based soft computing algorithm for tsp problems, Algorithms 7 (4) (2014) 663–684.

[357] Q. Xu, L. Guo, N. Wang, J. Pan, L. Wang, A novel oppositional biogeography-based optimization for combinatorial problems, ICNC (2014) 412–418.

[358] Q. Xu, L. Wang, B. He, N. Wang, Modified opposition-based differential evolution for function optimization, J. Comput. Inf. Syst. 7 (5) (2011) 1582–1591.

[359] Q. Xu, L. Wang, N. Wang, X. Hei, L. Zhao, A review of opposition-based learning from 2005 to 2012, Eng. Appl. Artif. Intell. 29 (2014) 1–12.

[360] M. Yaghini, M.M. Khoshraftar, M. Fallahi, Hiopga: a new hybrid metaheuristic algorithm to train feedforward neural networks for prediction, in Proceedings of the International Conference on Data Mining, pages 18-21, 2011.

[361] M. Yaghini, M.M. Khoshraftar, M. Fallahi, A hybrid algorithm for artificial neural network training, Eng. Appl. Artif. Intell. 26 (1) (2013) 293–301.

[362] C. Yang, J.-K. Zhang, L.-X. Guo, Investigation on the inversion of the atmospheric duct using the artificial bee colony algorithm based on opposition-based learning.

[363] G.-P. Yang, S.-Y. Liu, J.-K. Zhang, Q.-X. Feng, Control and synchronization of chaotic systems by an improved biogeography-based optimization algorithm, Appl. Intell. 39 (1) (2013) 132–143.

[364] L. Yang, S. Xijia, C. Deng, Opposition-based learning particle swarm optimization of running gait for humanoid robot, Int. J. Smart Sens. Intell. Syst. 8 (2) (2015).

[365] L. Yang, L. Zhou, A novel bp neural network for forecasting agriculture water consumption, in 2011 International Conference on Graphic and Image Processing, pages 828579-828579. International Society for Optics and Photonics, 2011.

[366] X. Yang, J. Cao, K. Li, P. Li, Improved opposition-based biogeography optimization. in Advanced Computational Intelligence (IWACI), 2011 Fourth International Workshop on, pages 642-647. IEEE, 2011.

[367] Y. Yang, Z. Cui, J. Wu, G. Zhang, X. Xian, Fuzzy c-means clustering and opposition-based reinforcement learning for traffic congestion identification, J. Inf. Comput. Sci. 9 (2012) 2441–2450.

[368] S. Yazdani, J. Shanbehzadeh, Balanced cartesian genetic programming via migration and opposition-based learning: application to symbolic regression, Genet. Program. Evol. Mach. 16 (2) (2015) 133–150.

[369] X. Youa, F. Haoc, Y. Maa, A hybrid differential evolution algorithm solving complex multimodal optimization problems?.

[370] S. Yu, S. Zhu, Y. Ma, D. Mao, Enhancing firefly algorithm using generalized opposition-based learning, Computing 97 (7) (2015) 741–754.

[371] Q. Yuan, G. Dai, The improved nsga-ii based on reverse learning mechanism. 2015.

[372] U. Yüzgeç, Performance comparison of differential evolution techniques on optimization of feeding profile for an industrial scale baker's yeast fermentation process, ISA Trans. 49 (1) (2010) 167–176.

[373] Q. Zhai, D. Yuan, H. Zhang, K. Gao, Parallelization of obl based pso k-means algorithm using opencl architecture, in Natural Computation (ICNC), 2014 10th International Conference on, pages 714-719. IEEE, 2014.

[374] C. Zhang, Z. Ni, Z. Wu, L. Gu, A novel swarm model with quasi-oppositional particle, in Information Technology and Applications, 2009. IFITA'09. International Forum on, volume 1, pp. 325–330. IEEE, 2009.

[375] X. Zhang, S.Y. Yuen, Opposition-based adaptive differential evolution, in Evolutionary Computation (CEC), 2012 IEEE Congress on, pp. 1–8. IEEE, 2012.

[376] F. Zhao, J. Zhang, J. Wang, C. Zhang, A shuffled complex evolution algorithm with opposition-based learning for a permutation flow shop scheduling problem, Int. J. Comput. Integr. Manuf. 28 (11) (2015) 1220–1235.

[377] L. Zhao, Q. Xu, J. Pan, Influence of jumping rate on opposition-based differential evolution using the current optimum, Inf. Technol. J. 12 (5) (2013) 959.

[378] P. Zhao, H. Li, Opposition-based cuckoo search algorithm for optimization problems, in Computational Intelligence and Design (ISCID), 2012 Fifth International Symposium on, volume 1, pages 344-347. IEEE, 2012.

[379] P.-J. Zhao, A hybrid harmony search algorithm for numerical optimization, in Computational Aspects of Social Networks (CASoN), 2010 International Conference on, pages 255-258. IEEE, 2010.

[380] X. Zhao, B. Yang, S. Gao, Y. Chen, Multi-contour registration based on feature points correspondence and two-stage gene expression programming, Neurocomputing 145 (2014) 512–529.

[381] Y. ZHONG, J. LIN, X. LIN, H. YANG, An improved particle swarm optimization algorithm for protein structure prediction based on ab model, J. Theor. Appl. Inf. Technol. 51 (1) (2013).

[382] J. Zhou, W. Fang, X. Wu, J. Sun, S. Cheng, An opposition-based learning competitive particle swarm optimizer, in Evolutionary Computation (CEC), 2016 IEEE Congress on, pp. 515–521. IEEE, 2016.

[383] J. ZHOU, H. WANG, W. SU, An opposition effective gsa based memetic algorithm for permutation flow shop scheduling.

[384] X. Zhou, Z. Wu, C. Deng, H. Peng, Enhancing artificial bee colony algorithm with generalised opposition-based learning, Int. J. Comput. Sci. Math. 6 (3) (2015) 297–309.

[385] X. Zhou, Z. Wu, H. Wang, Elite opposition-based differential evolution for solving large-scale optimization problems and its implementation on gpu, in Parallel and Distributed Computing, Applications and Technologies (PDCAT), 2012 Proceedings of the 13th International Conference on, pages 727-732. IEEE, 2012.

[386] X. Zhou, Z. Wu, H. Wang, S. Rahnamayan, Gaussian bare-bones artificial bee colony algorithm, Soft Comput. (2014) 1–18.